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Unconditionally stable time marching scheme for Reynolds stress models

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ABSTRACT

Progress toward a stable and efficient numerical treatment for the compressible Favre–Reynolds-averaged Navier–Stokes equations with a Reynolds-stress model (RSM) is presented. The mean-flow and the Reynolds stress model equations are discretized using finite differences on a curvilinear coordinates mesh. The convective flux is approximated by a third-order upwind biased MUSCL scheme. The diffusive flux is approximated using second-order central differencing, based on a full-viscous stencil. The novel time-marching approach relies on decoupled, implicit time integration, that is, the five mean-flow equations are solved separately from the seven Reynolds-stress closure equations. The key idea is the use of the unconditionally positive-convergent implicit scheme (UPC), originally developed for two-equation turbulence models. The extension of the UPC scheme for RSM guarantees the positivity of the normal Reynolds-stress components and the turbulence (specific) dissipation rate for any time step. Thanks to the UPC matrix-free structure and the decoupled approach, the resulting computational scheme is very efficient. Special care is dedicated to maintain the implicit operator compact, involving only nearest neighbor grid points, while fully supporting the larger discretized residual stencil. Results obtained from two- and three-dimensional numerical simulations demonstrate the significant progress achieved in this work toward optimally convergent solution of Reynolds stress models. Furthermore, the scheme is shown to be unconditionally stable and positive.

1. Introduction

Among the Reynolds-Averaged Navier–Stokes (RANS) turbulence models, Reynolds-stress models (RSM), also termed second-moment closure models, are perceived as the most advanced ones. Reynolds stress models are based on the six transport equations for the individual Reynolds-stress tensor components. A clear advantage of RSM (over first-order closure models) is that the production term does not require approximations. It is the production term that is primarily responsible for the anisotropy and the selective response of turbulence to different strain types. Hence, it is expected that this higher level of modeling, representing more elaborate physics, would be beneficial in terms of accurate flow predictions.

Despite their advantages, Reynolds stress models have not gained widespread acceptance and are mostly used by the academic community. Apparently, the two main reasons for this are poor numerical stability and the extra computer resources required compared to eddy-viscosity turbulence models. Several numerical issues can be clearly identified as the obstacles toward a robust implementation of Reynolds-stress models:

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- Incorporation of the Reynolds-stress tensor in the momentum equations.
- Preserving realizability.
- Low-diffusion approximate Riemann solvers.
- Stable time marching.

Two difficulties arise in the incorporation of the Reynolds stress tensor in the momentum equations. The first difficulty is a numerical one, known as the checkerboard problem. When the collocated grid approach is used, inappropriate interpolation of the Reynolds stress to evaluate the intermediate grid points may results in oscillatory solutions. This is attributed to the strong coupling of the mean velocity and the turbulent stress field. The second difficulty is associated with the analytical expression of the Reynolds stress tensor. Namely, when employing a linear eddy viscosity model, the linear Reynolds stress tensor is proportional to the (mean-flow) viscous stress tensor, contributing to the stabilization of the flow solver. In contrast, when using RSM, the Reynolds stress tensor may have a destabilizing effect, especially in diffusion-dominated regions. Several strategies were proposed in the past to overcome the problem of checkerboard oscillations. Huang and Leschziner [1] and Launder [2] proposed to use a staggered discretization of the Reynolds stresses. A special interpolation for evaluating the Reynolds stress tensor at the intermediate grid points was proposed by Obi et al. [3] and by Lai [4] for the collocated grid approach. In order to alleviate the inherent destabilizing effect of the Reynolds stress tensor in the momentum equation, Huang and Leschziner [1] proposed a special stabilization technique. They designed an apparent viscosity tensor which is derived directly from the Reynolds stress model. This apparent viscosity is incorporated into a Boussinesq-*like* term and a remaining term. The Boussinesq-*like* term is treated implicitly, thus enhancing the flow solver robustness.

Realizability was first addressed by Schumann [5]. The realizability constraints require that; a Reynolds stress model yield non-negative energy components, and that the off-diagonal components satisfy the Schwarz inequality. It has been long-known that a non-realizable Reynolds stress model can lead to numerical instabilities. Lumley [6,7] was the first to advocate the systematic use of realizability constraints in the design of Reynolds stress models. However, even if a model is designed to be fully realizable in the analytical sense, the numerical solution may still violate realizability constraints. An explicit procedure to enforce realizability was proposed by Chassaing et al. [8]. At each iteration, the realizability constraints were checked at every computational grid point, and in a case of a violation, the local solution of the Reynolds stress model was set to zero. Alternatively, Chaouat [9] enforced realizability for homogeneous flows in principle axes.

Low-diffusion approximate Riemann solvers are routinely used for the mean-flow equations, especially for compressible flow solvers. They are also used to approximate the convective flux of RANS turbulence models, usually based on the passive scalar approach [10]. However, it was found that the passive scalar approach may promote numerical instabilities [11,12], especially when low-diffusion Riemann solvers are used in Reynolds stress models. According to Ben Naser et al. [13], the numerical instabilities are attributed to the incorrect treatment of the contact discontinuity. To circumvent this problem, Ben Naser et al. proposed to use a hybrid flux, in which the convective flux of the mean-flow equations is approximated via a low-diffusion Riemann flux, while the convective flux of the Reynolds stress model is approximated by a more dissipative mass-flux, such as the Van-leer flux [14].

Only a few studies in the past proposed special time marching schemes for the Reynolds-averaged Navier–Stokes equations with Reynolds stress models. An efficient and robust implicit scheme has been proposed by Chassaing et al. [8], relying on the use of a dual time-stepping scheme (for steady-state flows), where the dual time steps are artificial time increments (LDTS). Thanks to the use of the LDTS algorithm and explicit enforcement of realizability constraints, they were able to obtain a robust flow solver. Moreover, Chassaing et al. [8] reported that thanks to the stabilization due to the LDTS algorithm, there was no further need to use the apparent viscosity idea [1]. More recently, Chaouat [15] proposed a special, split time-marching scheme where the convective and diffusive terms of the governing equations are integrated explicitly in time, while the Reynolds stress model source term is integrated implicitly. Specifically, Chaouat designed the implicit scheme to preserve the positivity of the turbulent dissipation and of the normal Reynolds stresses.

To date, only a few works on numerical methods for RSM have presented convergence plots. Moreover, in most of these works, the convergence was far from optimal, and it very seldom meets the acceptable standards of convergence with two-equation turbulence models, if at all. The insufficient convergence observed with RSM is sometimes misinterpreted as a result of physical oscillations, when in fact it may be related to numerical instabilities [8]. Therefore, there is a clear need for progress in developing convergent methods for RSM, in order to achieve reliable numerical solutions.

The present study focuses on designing a robust, implicit time marching scheme for the compressible Favre-averaged Navier–Stokes equations with a Reynolds stress model, specifically for the Reynolds stress model itself. The key idea is to extend the unconditionally positive-convergent implicit time integration scheme (UPC) that was originally developed for two-equation turbulence models [16,17], for use with Reynolds stress models. An appropriate extension of the UPC scheme would enable the use of an infinite time step for the Reynolds stress model solver. Moreover, although the original UPC scheme has been derived regardless of order of accuracy, so far it has been applied to low-order accuracy methods only. Specifically, it was applied to first-order upwind methods for the convective flux and central differencing, second-order, thin-layer approximation for the diffusive flux. In this work, the UPC scheme is applied, for the first time, to high-order accuracy methods: third-order biased MUSCL for the convective term, and second-order central differencing, based on a full-viscous stencil for the diffusive part. However, a straightforward implementation of the UPC scheme when using high-order spatial discretization methods would significantly increase the implicit operator bandwidth, which would have resulted in an expensive scheme. In this work, a simple modification is proposed, keeping the implicit operator of the UPC scheme nearly as