Journal of Computational Physics 261 (2014) 1–22

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An optimal penalty method for a hyperbolic system modeling the edge plasma transport in a tokamak

Philippe Angot, Thomas Auphan *, Olivier Guès

Aix Marseille Université, CNRS, LATP UMR 7353, 39 rue F. Joliot Curie, 13453 Marseille Cedex 13, France

ARTICLE INFO

Article history:
Received 24 February 2012
Received in revised form 2 November 2013
Accepted 17 December 2013
Available online 21 December 2013

Keywords:
Penalization method
Nonlinear hyperbolic system
Boundary layer
Finite volumes
Plasma transport
Tokamak

ABSTRACT

The penalization method is used to take account of obstacles, such as the limiter, in a tokamak. Because of the magnetic confinement of the plasma in a tokamak, the transport occurs essentially in the direction parallel to the magnetic field lines. We study a 1D nonlinear hyperbolic system as a simplified model of the plasma transport in the area close to the wall. A penalization which cuts the flux term of the momentum is studied. We show numerically that this penalization creates a Dirac measure at the plasma-limiter interface which prevents us from defining the transport term in the usual distribution sense. Hence, a new penalty method is proposed for this hyperbolic system. For this penalty method, an asymptotic expansion and numerical tests give an optimal rate of convergence without spurious boundary layer. Another two-fields penalization has also been implemented and the numerical convergence analysis when the penalization parameter tends to 0 reveals the presence of a boundary layer.

1. Introduction

A tokamak is a machine to study plasmas and the fusion reaction induced by the magnetic confinement. The plasma at high temperature (10^8 K, in the center) is confined in a toroïdal chamber thanks to a magnetic field. One of the main goals is to perform controlled fusion with enough efficiency to be a reliable source of energy. But, since the magnetic confinement is not perfect, the plasma is in contact with the wall. In order to preserve the integrity of the wall and to limit the pollution of the plasma, it is crucial to control these interactions. Obviously, if it was possible to simulate wall plasma interactions, it would be significantly easier to optimize the configuration.

Plasma models can be classified into three main classes. First, there are the single particle models where we compute the trajectory of each particle, but, as the number of particles in tokamak is of the order of 10^{20}, the computational cost is prohibitive. The kinetic models study the distribution function $f(\mathbf{x}, \mathbf{v}, t)$ which represents the density of particles of speed $\mathbf{v}$, located in $\mathbf{x}$, at the time t. The kinetic models have seven variables (in a three dimensional spatial domain) so the computational cost is still heavy. The fluid model is the most approximate one, since it considers that the plasma has the same behavior as a fluid and uses equations similar to Navier–Stokes equations. The fluid approximation seems to be verified for the scrape-off layer (temperature of the order of 10^4 K) whereas, in the center of the tokamak, a kinetic model is necessary.

To take into account the boundary conditions in the complex geometry of a tokamak, we can use volume penalty methods. These methods consist in embedding the original domain into a fictitious larger and simple domain and to modify the model equations outside the original domain so that the boundary conditions are verified. One advantage of these methods is that we don't need to use a mesh fitted to the geometry of the domain. But, as we shall see later, in addition

* Corresponding author.
E-mail addresses: angot@cmi.univ-mrs.fr (Ph. Angot), tauphan@cmi.univ-mrs.fr (T. Auphan), gues@cmi.univ-mrs.fr (O. Guès).

0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.12.037

to well-posedness issue, the penalty method adds a modeling error, which needs to be controlled. Such approaches have already been implemented successfully for elliptic and parabolic systems [1], for incompressible or compressible flows [3, 21].

In the sequel, we study, using a fluid approximation of the plasma, a simplified system of equations governing the plasma transport in the scrape-off layer parallel to the magnetic field lines. A penalty method has been introduced by Isoardi et al. [19], which gives interesting results. But the numerical study was incomplete and the fact that the momentum flux is cut inside the limiter may provide a Dirac measure next to the interface.

In this paper, Section 2 is devoted to a presentation of the toy model considered and Section 3 gives the finite volume scheme which is used for the numerical tests provided in Sections 4 and 5. In Section 4, after a numerical study of the penalization of Isoardi et al. [19], we modify the boundary conditions to ensure the well-posedness of the hyperbolic system and we study numerically another penalization which generates a boundary layer. In Section 5, we propose an optimal penalty method which is free of boundary layer, a theoretical result is stated for a slightly different problem. At the end of Section 5, the results of numerical tests are presented, with an extension to a two-sides limiter.

This work completes the first results presented by the authors in [2].

2. The model hyperbolic problem

At the center of the reactor, the transport along the field lines is almost free of constraint and fast enough to consider that at our time scale, physical quantities are constant along a magnetic field line. This is not the case in the scrape-off layer: magnetic field lines are intercepted by wall components (such as the limiter in TORE SUPRA). When the ion bumps into the limiter, a recombination process occurs and transforms the ion into a neutral particle which may be trapped into the limiter or re-injected in the plasma (and re-ionized later). In this paper, we consider a very simple model taking only into account the transport in the direction parallel to the magnetic field lines (see for example [19,28]). It is a one dimensional 2×2 nonlinear hyperbolic system of conservation laws for the particle density N and the particle flux Γ, which reads:

$$\begin{cases} \partial_t N + \partial_x \Gamma = S_N \\ \partial_t \Gamma + \partial_x \left(\dfrac{\Gamma^2}{N} + N \right) = S_\Gamma & (t,x) \in \mathbb{R}_*^+ \times]-L, L[. \\ \text{Initial conditions:} \quad N(0,.) = N_0 \quad \text{and} \quad \Gamma(0,.) = \Gamma_0 \end{cases} \tag{1}$$

Here, the boundaries of the domain $x = L$ and $x = -L$ correspond to the limiter ones, which are material obstacles for the fluid (see Fig. 1). In the right-hand side, S_N and S_Γ are given source terms. This hyperbolic system is similar to the 1-D isoentropic Euler equation with a linear pressure law. For sufficiently regular solutions, it can be written in the following non-conservative quasilinear form:

$$\partial_t \begin{pmatrix} N \\ \Gamma \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 - \frac{\Gamma^2}{N^2} & 2\frac{\Gamma}{N} \end{pmatrix} \partial_x \begin{pmatrix} N \\ \Gamma \end{pmatrix} = \begin{pmatrix} S_N \\ S_\Gamma \end{pmatrix} \quad (t,x) \in \mathbb{R}_*^+ \times]-L, L[. \tag{2}$$

We note in the sequel $M = M(\Gamma, N) := \frac{\Gamma}{N}$ the Mach number. The eigenvalues of the matrix

$$\begin{pmatrix} 0 & 1 \\ 1 - M^2 & 2M \end{pmatrix} \tag{3}$$

are $\Lambda_1 = M - 1$ and $\Lambda_2 = M + 1$, hence $\Lambda_1 < \Lambda_2$ and the system is strictly hyperbolic.

The boundary conditions. there is a difficulty with the choice of the boundary conditions for the system (1) that we describe now. From physical arguments, it follows that the domain (namely the scrape-off layer) is basically divided into two regions [28]:

- One region far from the limiter, the pre-sheath, where the plasma is neutral and the Mach number $M = \Gamma/N$ of the plasma satisfies $|M| \leqslant 1$.
- One region next to the limiter (in a thin layer called the sheath area, whose typical thickness is of the order of 10^{-5} m), where the electroneutrality hypothesis does not hold and we have $|M| > 1$. More precisely $M > 1$ close to $x = L$ and $M < -1$ close to the boundary $x = -L$.

At first glance, it could seem natural to prescribe $M = 1$ (resp. $M = -1$) as a boundary condition at $x = L$ (resp. $x = -L$) for the system, since the physical arguments imply that $M = \pm 1$ very close to the obstacle (Bohm criterion). These are exactly the boundary conditions which are chosen in [19]. However, in that case, since the eigenvalues are $\Lambda_1 = M - 1$ and $\Lambda_2 = M + 1$, it follows that, at the plasma-limiter interface, one eigenvalue is 0 (the boundary is characteristic) and the other one corresponds to an outgoing wave (it is also true at $x = -L$). Thus, the problem (1) does not satisfy the usual sufficient conditions for well-posedness, see [6,18,25]: the number of boundary conditions ($= 1$) is not equal to the number of incoming eigenvalues ($= 0$).