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Unified semi-analytical wall boundary conditions applied to 2-D incompressible SPH

A. Leroy [a,*], D. Violeau [a], M. Ferrand [b], C. Kassiotis [a]

[a] Saint-Venant Laboratory for Hydraulics, Université Paris-Est, 6 quai Watier, 78400 Chatou, France
[b] MFEE, EDF R&D, 6 quai Watier, 78400 Chatou, France

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ABSTRACT

This work aims at improving the 2-D incompressible SPH model (ISPH) by adapting it to the unified semi-analytical wall boundary conditions proposed by Ferrand et al. [10]. The ISPH algorithm considered is as proposed by Lind et al. [25], based on the projection method with a divergence-free velocity field and using a stabilising procedure based on particle shifting. However, we consider an extension of this model to Reynolds-Averaged Navier–Stokes equations based on the k–ϵ turbulent closure model, as done in [10]. The discrete SPH operators are modified by the new description of the wall boundary conditions. In particular, a boundary term appears in the Laplacian operator, which makes it possible to accurately impose a von Neumann pressure wall boundary condition that corresponds to impermeability. The shifting and free-surface detection algorithms have also been adapted to the new boundary conditions. Moreover, a new way to compute the wall renormalisation factor in the frame of the unified semi-analytical boundary conditions is proposed in order to decrease the computational time. We present several verifications to the present approach, including a lid-driven cavity, a water column collapsing on a wedge and a periodic schematic fish-pass. Our results are compared to Finite Volumes methods, using Volume of Fluids in the case of free-surface flows. We briefly investigate the convergence of the method and prove its ability to model complex free-surface and turbulent flows. The results are generally improved when compared to a weakly compressible SPH model with the same boundary conditions, especially in terms of pressure prediction.

1. Introduction

Modelling incompressible flows with the Smoothed Particle Hydrodynamics (SPH) method has classically been done through weakly compressible SPH (WCSPH) models, as is thoroughly described in [33]. In this case, the pressure is calculated through an artificial equation of state, which causes the pressure prediction to be noisy and, in many cases, inaccurate. To remedy this issue, truly incompressible approaches were developed in the framework of SPH. In particular, Cummins and Rudman [5] adapted the projection method of Chorin [3,4] to SPH by solving a discrete Poisson equation for pressure, leading to an incompressible SPH method (ISPH). Comparisons between ISPH and WCSPH were done by Lee et al. [22], which showed that ISPH makes it possible to reduce the computational time while providing a better description of the pressure field than WCSPH. Several versions of the SPH projection method were proposed, the main three of them being:

* Corresponding author. Tel.: +33 (0)6 67 88 92 13.
E-mail addresses: agnes.leroy@edf.fr (A. Leroy), damien.violeau@edf.fr (D. Violeau), martin.ferrand@edf.fr (M. Ferrand), christophe.kassiotis@enpc.fr (C. Kassiotis).

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(i) the one proposed by Cummins and Rudman, which consists in maintaining zero divergence velocity, (ii) the one proposed by Shao and Lo [41], which consists in keeping density invariance and (iii) the one proposed by Hu and Adams [17], based on combining the two previously mentioned methods and thus solving two Poisson equations. In 2009, Xu et al. [49] made a comparative study between those methods and showed that each of them presented drawbacks. According to the latter authors, imposing the density invariance leads to noisy pressure fields, while imposing the zero velocity divergence gives very smooth pressure fields but leads to instabilities due to particle clustering. On the other hand, the method proposed by Hu and Adams [17], though being stable and providing smooth pressure fields, suffers from very high computational times. Thus, Xu et al. [49] proposed a stabilising method for the ISPH model based on keeping divergence-free velocity field, which makes it possible to accurately estimate the pressure while keeping computational time smaller than WCSPH. This method consists in slightly shifting the position of the particles at each iteration so as to avoid highly anisotropic particle spacing. The hydrodynamic variables are then corrected by adding the advection term corresponding to the position shift. This method was improved by Lind et al. [25], who proposed an expression for the position shift based on Fick's law of diffusion. They also extended the shifting method to free-surface flows. The algorithm proposed by Lind et al. [25] seems stable and able to accurately model a great variety of complex flows. Yet, the problem of the pressure wall boundary conditions remains.

A classical way of imposing wall boundary conditions in SPH is the imposition of repulsive forces such as the Lennard-Jones potential [33] or Monaghan and Kajtar's method [34]. These methods are easy to implement even for complex geometries and are computationally cheap, but lead to spurious behaviour, as pointed out by Ferrand et al. [10]. In particular, the fluid does not remain still near the walls in a hydrostatic case. Besides, they make it difficult – if not impossible – to accurately prescribe a Neumann pressure wall boundary condition, which is a serious issue for ISPH. Most available ISPH models in the literature are thus based on ghost particles [41,18,23] or mirror particles [14], for example in [22,49,25]. These two techniques are widely used to impose wall boundary conditions in SPH. However, they present serious drawbacks. First, ghost particles are not easy to place in case of complex geometries, especially in three dimensions. Moreover, for nearly all the existing ISPH models combined to ghost or mirror particle methods, a homogeneous Neumann wall boundary condition is imposed on the pressure [22,49,25]. This is done by manipulating the relevant entries in the linear system so that the value of the pressure is mirrored across the solid boundary. This is not an exact prescription of Neumann pressure wall boundary condition, and is a serious issue since the proper imposition of pressure boundary condition is crucial when solving the pressure Poisson equation. Yildiz et al. [50] proposed a new method for placing the ghost particles which seemed to improve the accuracy of the imposition of wall boundary condition, but still not exact, and their condition remained homogeneous. However, in many cases the pressure gradient at a solid wall is non-zero, so that a homogeneous boundary condition is not appropriate. In [16], Hosseini et al. tested a rotational projection scheme in SPH which makes it possible to impose a non-homogeneous Neumann pressure boundary condition by imposing a homogeneous boundary condition on the dynamic pressure. However, this technique does not make it possible to impose arbitrary non-homogeneous boundary condition.

Recently, other methods were proposed to model solid boundaries that account for the kernel truncation close to the wall, through the use of a wall renormalisation factor in the SPH discrete interpolation. Kulasegaram et al. [20] and De Leffe et al. [24] proposed approximate methods to calculate the renormalisation factor while Feldman and Bonet [9] proposed an analytical method for simple wall shapes. In these works, the application of the renormalisation factor in the discrete SPH interpolation formula led to the application of a boundary force in the Navier–Stokes equations. In [10], Ferrand et al. proposed a different way of computing the renormalisation factor together with a new formulation of the differential operators. This formulation is similar to the one Kulasegaram et al. proposed for the pressure gradient, but the boundary terms are properly represented for all the differential operators. In this framework, the imposition of boundary conditions can be done in a natural way through the boundary term of the new Laplacian operator. This was applied in [10] to the k–ϵ turbulence model where Neumann boundary conditions could be prescribed exactly on k and ϵ for the first time in SPH, the condition on ϵ being non-homogeneous. With this method the estimation of the fields is very accurate, even close to the walls. Associating the wall boundary conditions proposed by Ferrand et al. [10] to an ISPH model would make it possible to impose exactly arbitrary Neumann (or Dirichlet) boundary conditions on the pressure, and thus to properly model flows involving complex boundary geometries while taking advantage of the ISPH method. From now on these boundary conditions will be referred to as USAW boundary conditions (unified semi-analytical wall boundary conditions).

Recently, Macià et al. [27] applied the USAW boundary conditions to ISPH, but they focused on the prescription of Dirichlet boundary conditions on the pressure field, which is not appropriate in dynamic cases. Moreover, they did not present any applications of their ISPH model to 2-D or 3-D. In the present work an ISPH model is developed, in which exact arbitrary Neumann boundary conditions can be prescribed when solving the pressure Poisson equation in 2-D.

We will first describe the SPH interpolation in the frame of USAW wall boundary conditions. Then, the new ISPH model will be explained. We will see how the algorithm proposed by Lind et al. [25] can be adapted to the new boundary description, and see how to impose a non-homogeneous Neumann pressure boundary condition. We include laminar and turbulent (Reynolds-averaged) flows in the same framework, our purpose being to unify all wall boundary treatment from [10], including the Poisson equation and the k–ϵ model. In this way our method can deal with basic industrial turbulent flows with a reasonable quality of predictions, though the RANS (Reynolds-Averaged Navier–Stokes) approach is rather crude compared to LES (Large Eddy Simulation) models. Note that the k–ϵ model was applied for the first time to SPH by Violeau [44] and by Shao [40], but in these works the boundary conditions were not imposed properly on the turbulent fields.