Journal of Computational Physics 260 (2014) 222–248

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Accurate numerical modeling of 1D flow in channels with arbitrary shape. Application of the energy balanced property

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ARTICLE INFO

Article history:
Received 2 September 2013
Received in revised form 12 December 2013
Accepted 17 December 2013
Available online 24 December 2013

Keywords:
Channels
Arbitrary shape
Well-balanced
Roe solver
Shallow water
Energy balanced

ABSTRACT

This work focuses on the numerical treatment of 1D flow in channels with arbitrary shape using energy balanced arguments. The system of equations is defined using the mass and momentum conservation equations, allowing the resolution of hydraulic jumps where energy conservation arguments are not valid. When necessary, conservation of mechanical energy takes part actively in the numerical scheme when evaluating the source terms. The numerical scheme is based on an augmented Roe solver that involves the presence of source terms by adding an extra stationary wave. The characteristics of the numerical scheme include the energy balanced property, and being only first order accurate in time and space, leads to exact numerical solutions for steady solutions with independence of the grid refinement in channels with general geometries. Riemann problems considered here involve non-prismatic channels, bed variations and the resonance regime, including the limiting situation when the Riemann data belong to the resonance hypersurface. Numerical results point out that the finite volume numerical scheme with nonconservative terms presented here, converges to the exact solution. The well balanced property is ensured, as it is a particular case of the energy balanced property in cases of quiescent equilibrium.

1. Introduction

Riemann solver-based techniques have proved a successful tool for numerical simulation of conservation laws. Their extensions to ensure well balanced solutions [1,2] in the context of the shallow water equations have led to a wide number of well-balanced numerical schemes based on the preservation of motionless steady state in presence of irregular geometries [3–16]. The source terms modify the behavior of the solution so that the approximate solvers derived for the case without source terms are no longer valid. Then, when solving shallow flows in realistic applications, even in cases of smooth variations of topography, it is necessary to use different weak or approximate solutions. In practice, it is usual to find large gradients in the bed level and strong contractions or expansions in the basin geometry. Depending on the numerical approximations made, the estimation of the source terms may lead to gross errors for both steady and unsteady flows. Therefore, it is desirable to find approximate solvers that include the presence of source terms, but also it is of importance that these new solvers can clearly explain the impact of the evaluation of the source term in the approximate solution.

When dealing with hyperbolic systems with nonconservative terms, not only well balanced solutions must be ensured. In cases where quiescent equilibrium is not demanded, the analysis of the convergence to the exact solution as the mesh size tends to zero is of importance [17,18,10]. The definition of appropriate numerical schemes can be envisaged using families of paths connecting the left and right states of the Riemann problem, according to the theory of Dal Maso, LeFloch, and Murat [19]. The definition of appropriate families of paths is not an easy task, as nonlinear hyperbolic system with source

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0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.12.040

terms are not strictly hyperbolic and the total number of waves in a solution can possibly be larger than the number of characteristic fields as waves associated with a given family can be repeated. Also, it is possible to find cases with multiple solutions [10]. In the case of the resonance problem, weak solutions may depend both on the family of paths and the numerical scheme itself [18]. As a consequence, numerical strategies based on a direct discretization of the source terms are justified.

The effect of the numerical approaches made in the estimation of the source terms was analyzed in [20] in the context of two augmented Riemann solvers. Both augmented solvers, ARoe (Augmented Roe) [21] and HLLS [22] included the extra wave associated to the presence of bed discontinuity. The former was based on the upwind discretization of the source terms in [1] and the Roe solver [23] defined for the homogeneous case. The later, named HLLS was constructed by including an extra wave in the original HLL [24]. Also, the HLLC solver [25] was extended in [22] to include the presence of the contact wave and the extra wave associated to the presence of bed discontinuity. This family of augmented solvers provides a definition of the inner states arising from a Riemann problems (RP). While in the homogeneous case, the Roe or the HLL solvers define a characteristic constant U* in the initial discontinuity, the ARoe and HLLS solvers provide two different values for the solution depending on the side of the solution plane. In this way, the well balanced property is ensured, as the evolution of the approximate solution ensures continuity of the initial values, including the inner states, in each side of the RP.

The well balanced property is desirable when constructing a numerical scheme but it does not explain nor prevents the appearance of negative values of water depth in wet/dry or even wet/wet Riemann problems. This property is a particular result for quiescent equilibrium and does not provide information for more general cases. Also, well balanced schemes include the definition of a suitable estimation of the source terms, otherwise, they fail. Only when the complete description of the approximate solver used is provided, it is possible to analyze correctly the performance of the numerical scheme associated to that solver. The description of the approximate solver allows to analyze the effect of the estimations made over the source terms and correct them when necessary. This procedure is not far away from what is actually done when solving conservation laws in the homogeneous case. For instance, the Roe scheme requires an entropy fix in transcritical problems [26]. Entropy fixes avoid the appearance of unphysical solutions in the inner states of the RP.

Augmented approximate solvers have proved a successful tool in free surface flows. In [27] a novel ARoe solver was presented to solve the 2D shallow water equations including variable density over discontinuous bed. The definition of the inner states allowed to propose correction rules over reactive source term integrals. Simulation of mud/debris floods provided accurate results in [28], where the effect of frictional source terms in the approximate solution was studied. In that work the well balanced included cases with quiescent equilibrium in uneven free surfaces and a correct managing of starting/stopping flow conditions. In [20] energy balanced numerical schemes based in the HLLS and ARoe solvers were presented in the context of the 2D shallow water equations in presence of bed variations. A suitable integration of the source term allowed to reproduce exactly steady solutions with independence of the mesh refinement and ensured convergence to the solution in resonant RP's. Following earlier works [3,4], where especial attention was paid to a correct treatment of the geometrical source terms arising from the changes in cross-sectional areas, the ARoe solver presented in [20] is extended here to include the source terms arising from the presence of arbitrary channel geometries and variable topography.

This work is organized as follows. In Section 2 the mathematical model of the 1D shallow water equations is presented. Considering that the numerical modeling in presence of arbitrary channel geometries involves pressure forces that may be complex to estimate, the shallow water equations are written in terms of differences, reducing notably the computational effort. This allows to define a Jacobian matrix that is the basis of the approximate solver presented in Section 3. As the system of equations is written in terms of differences of the conserved variables, the numerical scheme is written in terms of flux differences. In Section 4 the details of the numerical approximations made over the source terms to ensure the energy balanced property are provided. Section 5 is devoted to numerical results including steady and unsteady problems.

2. Mathematical model

One-dimensional shallow water flows of practical application in hydraulics such as river flows and channels can be modeled using mass and momentum conservation laws [3,4]

$$\begin{aligned} &\frac{\partial A}{\partial t} + \frac{d}{dx} Q = 0 \\ &\frac{\partial Q}{\partial t} + \frac{d}{dx}\left(\frac{Q^2}{A} + gI_1\right) = g\left[I_2 + A(S_0 - S_f)\right] \end{aligned} \tag{1}$$

where Q is the discharge, A is the wetted cross section, g is the acceleration of gravity and S_0 is the bed slope. I_1 represents the hydrostatic pressure force term

$$I_1(x, A) = \int_0^{h(x,A)} \left[h(x, A) - \eta\right]\sigma(x, \eta)\, d\eta \tag{2}$$