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A numerical method for a model of two-phase flow in a coupled free flow and porous media system

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ABSTRACT

In this article, we study two-phase fluid flow in coupled free flow and porous media regions. The model consists of coupled Cahn–Hilliard and Navier–Stokes equations in the free fluid region and the two-phase Darcy law in the porous medium region. We propose a Robin–Robin domain decomposition method for the coupled Navier–Stokes and Darcy system with the generalized Beavers–Joseph–Saffman condition on the interface between the free flow and the porous media regions. Numerical examples are presented to illustrate the effectiveness of this method.

1. Introduction

We study two-phase fluid flow in coupled free flow and porous media regions. The problem arises in several industrial applications, such as well-reservoir coupling in petroleum engineering, fluid–organ interactions and industrial filtering where a fluid passes through a filter to remove unwanted particles. More precisely, we consider the following situation: incompressible two-phase fluid in a region Ω_f flows across an interface Γ_I into a porous medium domain Ω_p as described in Fig. 1. The model we use consists of coupled Cahn–Hilliard and Navier–Stokes equations for the free flow region Ω_f and a two-phase Darcy law for the two-phase flows in the porous medium region Ω_p. There are many previous works addressing single-phase Stokes–Darcy system of single-phase flows [1–4] and the related problems such as Stokes–Navier Stokes systems [5,6], Stokes–Laplacian systems [7,8]. However there are very few work for the two-phase flow in such a coupled setting.

For the two-phase flow, the moving contact line problem arises when interface intersects the solid wall. In [9,10], a coupled system of the Cahn–Hilliard (CH) equations and the Navier–Stokes (NS) equations with the generalized Navier boundary condition (GNBC) is proposed. It is shown that the numerical results based on the GNBC can reproduce quantitatively the results from the Molecular Dynamics (MD) simulation. This indicates that the new model can accurately describe the behavior near the contact line. The above phase field model with GNBC has been used to model several moving contact line problems successfully [11,12]. Several efficient numerical methods were also developed in recent years for the coupled Cahn–Hilliard and Navier–Stokes equations with generalized Navier boundary condition [13–15].

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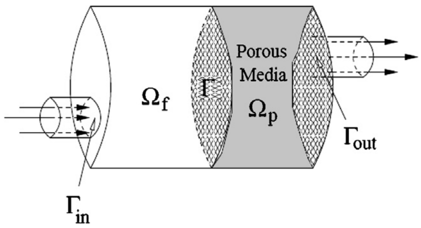

Fig. 1. Illustration of flow domain.

In this paper, we propose a transmission condition for the two phase flow at the interface boundary of the free flow region and the porous media region. For the single phase flow, the Beavers–Joseph–Saffman (BJS) [16–18] condition states that the tangential slip velocity at the interface is proportional to the shear stress at the interface:

$$\mathcal{L}_p^{-1}u = -\nu \frac{\partial u}{\partial n}$$

where $\mathcal{L}_p$ is related to the permeability and the porosity of the porous media. For the two phase flow, we expect that the slip velocity is proportional to the total stress at the fluid/porous media interface which, in addition to the fluid shear stress, should also consists of Young stress. Therefore a GNBC type of fluid/porous media interface condition (2.21) is proposed in this paper which is a natural generalization of the BJS condition. We will call this interface condition the generalized Beavers–Joseph–Saffman (GBJS) condition.

We then develop a numerical method for the coupled system of the two-phase Darcy law for the two-phase flows in the porous medium region and the Cahn–Hilliard and Navier–Stokes equations for the free flow region with the generalized BJS condition. We consider Implicit Pressure Explicit Saturation (IMPES) treatment for the Darcy region. The IMPES method was originally developed by Sheldon et al. [19] and Stone and Gardner [20]. The basic idea of this classical method for solving a partial differential coupled system for two-phase flow in a porous medium is to separate the computation of pressure from that of saturation. Namely, this coupled system is split into a pressure equation and a saturation equation, and the pressure and saturation equations are solved using implicit and explicit time approximation approaches, respectively. Based on the fact that the pressure changes less rapidly in time than the saturation in this two-phase flow system, Chen et al. developed an IMPES method for solving a partial differential system for two-phase flows in porous medium in [21]. This method takes a larger time step for the pressure than for the saturation to save computational time. In [22], it is shown that the IMPES method is equivalent to a two-stage preconditioner when solving this coupled system. In the free flow region, we solve the coupled CH-NS system with a finite element method in space and a second-order semi-implicit scheme in time. In this method [15], the Cahn–Hilliard equations and Navier–Stokes equations are solved in a decoupled way. For the Cahn–Hilliard equations, a convex temporal splitting scheme and P1-P1 finite element spatial discretization method are employed. In the porous medium region, the improved IMPES method is used to solve the Darcy law for the two-phase flows. We then combine the techniques described above to solve the coupled Navier–Stokes and Darcy two-phase flow system. A Robin–Robin domain decomposition method is employed to solve the two systems iteratively.

The rest of the paper is organized as follows. We describe the modeling equations and the transmission conditions in Section 2. In Section 3, after a brief introduction of the numerical method for CH-NS system and IMPES method, we present the Robin–Robin domain decomposition method with GNBC transmission condition. Numerical examples together with some numerical analysis are presented in Section 4. A conclusion is put in Section 5. Appendix A gives an equivalence analysis between the coupled CH-NS and Darcy system and the decoupled CH-NS and Darcy system with Robin boundary conditions.

2. Modeling equations

Let Ω_f and Ω_p denote bounded Lipschitz domains for the free flow region and the porous media region, respectively. The interface boundary between the domains is denoted by $\Gamma_I := \partial\Omega_f \cap \partial\Omega_p$. Let $\Omega := \Omega_f \cup \Omega_p \cup \Gamma_I$. The outward unit normal vectors to Ω_f and Ω_p are denotes by $\mathbf{n}_f$ and $\mathbf{n}_p$, respectively. The tangent vectors on Γ_I are denoted as τ (for $n=2$), or $\tau_l,\ l=1,2$ (for $n=3$).

We assume that there is an inflow boundary Γ_{in}, a subset of $\partial\Omega_f\backslash\Gamma_I$, which is separated from Γ_I and an outflow boundary Γ_{out}, a subset of $\partial\Omega_p\backslash\Gamma_I$ which is also separated from Γ_I. See Fig. 1 for an illustration of the domain of the problem. Define $\Gamma_f := \partial\Omega_f\backslash(\Gamma_I \cup \Gamma_{in})$, and $\Gamma_p := \partial\Omega_p\backslash(\Gamma_I \cup \Gamma_{out})$.

2.1. The coupled Cahn–Hilliard and Navier–Stokes equations in the free flow region

In Ω_f, we assume that the two flow is governed by a coupled system of Cahn–Hilliard Navier–Stokes equations, subject to a specified flow rate, q_f, across Γ_{in}. For simplicity we consider here the case of a single inflow boundary Γ_{in}.

$$\frac{\partial \phi}{\partial t} + \mathbf{u}\cdot\nabla\phi = M\Delta\mu, \tag{2.1}$$