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Adaptive discontinuous evolution Galerkin method for dry atmospheric flow

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ABSTRACT

We present a new adaptive genuinely multidimensional method within the framework of the discontinuous Galerkin method. The discontinuous evolution Galerkin (DEG) method couples a discontinuous Galerkin formulation with approximate evolution operators. The latter are constructed using the bicharacteristics of multidimensional hyperbolic systems, such that all of the infinitely many directions of wave propagation are considered explicitly. In order to take into account multiscale phenomena that typically appear in atmospheric flows, nonlinear fluxes are split into a linear part governing the acoustic and gravitational waves and a nonlinear part that models advection. Time integration is realized by the IMEX type approximation using the semi-implicit second-order backward differentiation formula (BDF2). Moreover in order to approximate efficiently small scale phenomena, adaptive mesh refinement using the space filling curves via the AMATOS function library is employed. Four standard meteorological test cases are used to validate the new discontinuous evolution Galerkin method for dry atmospheric convection. Comparisons with the Rusanov flux, a standard one-dimensional approximate Riemann solver used for the flux integration, demonstrate better stability and accuracy, as well as the reliability of the new multidimensional DEG method.

1. Introduction and meteorological motivation

A characteristic property of atmospheric flows is their multiscale nature with wave speeds differing by orders of magnitude. If the Mach and Froude numbers are small, the acoustic and gravitational waves are much faster than advection, but only the latter is of primary interest for numerical weather prediction. Naive explicit time integration would yield prohibitively expensive numerical simulations, which makes a suitable splitting of fast and slow waves highly desirable. This idea is not new and has been used extensively in previous meteorological simulations. Many operational nonhydrostatic weather models use split-explicit methods, where different time steps are used for slow and fast waves, respectively, cf. [16,26], the National Center for Atmospheric Research [25], Pennsylvania State University/National Center for Atmospheric Research [49] and the German Weather Service [41]. Another common approach is based on semi-implicit time discretization; here the fast waves that are of less interest are approximated implicitly, whereas slow advection is treated explicitly. Several methods following this idea can be found, e.g., in [2,10,16,23,24,34,35,39,47] to name just a few.

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Another characteristic of many atmospheric flows is their multidimensional character with different localized structural phenomena such as, e.g., the cloud-environment interface. A convenient tool to approximate these local structures efficiently is mesh adaptivity. Indeed, adaptive mesh refinement has been applied in the atmospheric sciences quite successfully over the past decades, see, e.g. [3,6,42]. Of course, the strategy where and how the mesh has to be refined is a difficult scientific problem and depends on the particular application. The final application we have in mind is the simulation of an evolving cumulus cloud and its interaction with the environment. This is an important meteorological problem, since the evaporative cooling at the cloud-environment boundary and its impact on the cloud evolution are not well understood [21,34]. Consequently, efficient adaptive numerical schemes can be expected to improve the insight into the underlying physical processes by explicitly resolving the interplay between the larger scales of the cloud environment and the smaller scales inside the cloud and at its boundary. In order to approximate localized structures efficiently, we will work with adaptive meshes using the space filling curves via the AMATOS function library, cf. [5].

In this paper we develop a new semi-implicit genuinely multidimensional method within the framework of the discontinuous Galerkin method. The method is implemented in the discontinuous Galerkin solver by Giraldo and Warburton [17], see also recent results [33,34] for applications to the Euler equations. However, instead of the Rusanov flux (a standard one-dimensional approximate Riemann solver), that has been used in [33,34], the flux integration within the discontinuous Galerkin method is now realized by means of a genuinely multidimensional evolution operator. The latter is constructed using the theory of bicharacteristics in order to take all infinitely many directions of wave propagation into account. The approximate evolution operator can be interpreted as a multidimensional numerical flux function. In the finite volume framework the finite volume evolution Galerkin (FVEG) method has been used successfully for various physical applications, e.g., wave propagation in heterogeneous media [1], the Euler equations of gas dynamics [7,30] and the shallow water equations [12,22,27]. In [32] we derived exact integral representation and approximate evolution operators for three-dimensional hyperbolic conservation laws and presented the results of the FVEG method for the acoustic equation. The FVEG method has been shown to be more accurate than standard FV methods based on the one-dimensional Riemann solver, see also, [28] for further references. In order to illustrate high accuracy, stability and robustness also for the new DEG method we will concentrate on two-dimensional dry atmospheric flows and standard meteorological test cases. Generalization to fully three-dimensional meteorological test cases is the subject of our future study. To improve efficiency of the new DEG scheme on adaptive triangular grids we have also studied its performance when the multidimensional evolution operator is ported to GPUs. In our recent paper [7] we have shown that we can achieve a considerable speedup of 30 (in comparison to a single CPU core) for the calculation of the evolution Galerkin operator for standard meteorological tests cases.

The remainder of the paper is organized as follows. In the next section we describe the mathematical model governing dry atmospheric flow, in Section 3 we derive the discontinuous Galerkin method for spatial discretization and the IMEX type approximation for time discretization. An emphasis is put on a non-standard discretization of the cell interface fluxes by means of multidimensional EG operators, cf. Section 3.2. In Section 4 we present the numerical experiments and illustrate the high accuracy and stability of the new EG method and show comparisons with the DG method that uses the standard Rusanov numerical flux.

2. Mathematical model

We start with the description of the mathematical model. Motion of compressible flows is governed by the Euler equations

$$\begin{aligned}
&\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\
&\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} + p\,\mathrm{Id}) = -\rho g \mathbf{k} \\
&\partial_t (\rho\theta) + \nabla \cdot (\rho\theta \mathbf{u}) = 0,
\end{aligned} \tag{2.1}$$

where ρ denotes the density, $\mathbf{u}$ velocity, p pressure and θ the potential temperature. Further, g represents the gravitational constant, Id is the identity matrix and $\mathbf{k}$ the unit vector in the vertical direction. Denoting T temperature, the potential temperature can be obtained from the equation of adiabatic process in an ideal gas

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p}, \quad R = c_p - c_v.$$

We use potential temperature as a variable since it is better suited for generalization to moist atmospheric flow. In order to close the system we determine pressure from the state equation

$$p = p_0\left(\frac{R\rho\theta}{p_0}\right)^{\gamma},$$

where $\gamma = c_p/c_v$ is the adiabatic constant and $p_0 = 10^5$ Pa the reference pressure.