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Low-diffusion approximate Riemann solvers for Reynolds-stress transport

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ABSTRACT

The paper investigates the use of low-diffusion (contact-discontinuity-resolving) approximate Riemann solvers for the convective part of the Reynolds-averaged Navier–Stokes (RANS) equations with Reynolds-stress model (RSM) for turbulence. Different equivalent forms of the RSM–RANS system are discussed and classification of the complex terms introduced by advanced turbulence closures is attempted. Computational examples are presented, which indicate that the use of contact-discontinuity-resolving convective numerical fluxes, along with a passive-scalar approach for the Reynolds-stresses, may lead to unphysical oscillations of the solution. To determine the source of these instabilities, theoretical analysis of the Riemann problem for a simplified Reynolds-stress transport model-system, which incorporates the divergence of the Reynolds-stress tensor in the convective part of the mean-flow equations, and includes only those nonconservative products which are computable (do not require modelling), was undertaken, highlighting the differences in wave-structure compared to the passive-scalar case. A hybrid solution, allowing the combination of any low-diffusion approximate Riemann solver with the complex tensorial representations used in advanced models, is proposed, combining low-diffusion fluxes for the mean-flow equations with a more dissipative massflux for Reynolds-stress-transport. Several computational examples are presented to assess the performance of this approach, demonstrating enhanced accuracy and satisfactory convergence.

1. Introduction

Current trends in RANS CFD (Reynolds-averaged Navier–Stokes computational fluid dynamics) for complex aircraft configurations [1] aim at developing methods of high predictive accuracy [2]. From a turbulence modelling point-of-view, this requires the combination of advanced anisotropy-resolving closures [3] for the Reynolds-stresses (which appear in the averaged mean-flow equations) with transport-equation closures for transition [4]. Regarding the fully turbulent part of the flow model, differential second-moment closures (SMCs) or synonymously Reynolds-stress models (RSMs) have the advantage of treating terms representing the influence of turbulence on the mean-flow ($\bar{\rho} r_{ij} := \overline{\rho u_i'' u_j''}$, where ρ is the density, u_i are the velocity components in the Cartesian system with space-coordinates x_i, $\bar{\cdot}$ denotes Reynolds-averaging, and $\cdot''$ denotes Favre fluctuations) as variables of the system of PDEs describing the flow, in this way transferring the drawbacks of a posteriori performance of algebraic closures [5] to other correlations appearing in the Reynolds-stress model (velocity/pressure-gradient $\Pi_{ij} := -\overline{u_i' \partial_{x_j} p'} - \overline{u_j' \partial_{x_i} p'}$ where p is the pressure and $\cdot'$ denotes Reynolds fluctuations, diffusion by

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triple velocity-correlations $d_{ij}^{(u)} := -\partial_{x_\ell}\overline{\rho u_i'' u_j'' u_\ell''}$ where repeated indices imply the Cartesian-tensor summation convention [6, pp. 644–645], anisotropy of the rate-of-dissipation tensor $\varepsilon_{ij} - \frac{2}{3}\varepsilon\delta_{ij}$ where $\varepsilon := \frac{1}{2}\varepsilon_{\ell\ell}$ is the dissipation-rate of turbulent kinetic energy and δ_{ij} is Kronecker's δ [7, p. 10]). On the other hand, the numerically reassuring concept of eddy-viscosity, which introduces only minor modifications in the mean-flow equations, is lost. Incidentally, eddy-viscosity is not a physically definable quantity in general inhomogeneous flows with complex strains.

Most RSM–RANS solvers, both structured [8–10] and unstructured [11,12], apply variables-reconstruction [13] to define left (L) and right (R) states at cell-interfaces [14], which determine fluxes by the approximate solution of the corresponding Riemann problem [15]. We loosely include in the term approximate Riemann solver (ARS) different approaches used in defining the numerical flux, i.e. approximate Riemann solvers [16,17], flux-difference-splitting [18], and flux-splitting [19,20]. Early results on laminar boundary-layer flow [21] using $O(\Delta\ell)$ reconstruction ($\Delta\ell$ is the largest distance between the vertices of the grid-cell), have shown that some fluxes are more dissipative than others, in the sense that they introduce more numerical diffusion, especially in flows dominated by shear (boundary-layers, jets and wakes). It is well known [22,23] that high-order reconstruction of low-level (diffusive) fluxes results in high-order-accurate schemes. Therefore, differences between fluxes observed for $O(\Delta\ell)$ reconstruction are less pronounced when higher-order reconstruction is used, but may influence the rate of grid-convergence with grid-refinement. The improvement in flow prediction by using numerical fluxes which correctly resolve contact-discontinuities of the associated Riemann problem was demonstrated by the construction of the HLLC ARS [17], compared to the HLL ARS [16]. Batten et al. [24] categorize approximate Riemann solvers with respect to the fidelity with which they reproduce the structure of the solution of the Riemann problem. From this point-of-view, 4-state solvers for the Euler equations, like the HLLC ARS [17] with appropriate choice of the wavespeeds [25], are obviously contact-discontinuity-resolving. This analysis cannot be applied to all types of fluxes (e.g. flux-splitting [19,20]). In a more general context, the term contact-discontinuity-resolving follows from the work of Liou [26], who suggested a rigorous definition of what is meant by low-diffusion numerical fluxes. Consider the Euler equations [15, pp. 102–111], with conservative variables $\underline{u}_{\text{E}} := [\rho, \rho u, \rho v, \rho w, \rho e_t]^{\text{T}}$ and flux $\underline{\vec{F}}_{\text{E}}(\underline{u}) \cdot \vec{e}_n$ in the direction of the unit-vector $\vec{e}_n$, for which the numerical dissipation of the massflux $F_\rho^{\text{NUM}}(\underline{u}_{\text{E}}^{\text{L}}, \underline{u}_{\text{E}}^{\text{R}}; \vec{e}_n)$, defined with respect to an average flux $F_\rho^{\text{AVG}}(\underline{u}_{\text{E}}^{\text{L}}, \underline{u}_{\text{E}}^{\text{R}}; \vec{e}_n)$ [27], $\frac{1}{2}\mathcal{D}_\rho(\underline{u}_{\text{L}}, \underline{u}_{\text{R}}; \vec{e}_n) := F_\rho^{\text{AVG}} - F_\rho^{\text{NUM}}$ [27, (28), p. 5], is expanded as $\mathcal{D}_\rho = D_{\rho,\rho}\Delta_{\text{LR}}\rho + \sum_{\ell=1}^{3}\mathcal{D}_{\rho,\ell}\Delta_{\text{LR}}u_\ell + \mathcal{D}_{\rho,p}\Delta_{\text{LR}}p$ with respect to the differences $\Delta_{\text{LR}}(\cdot) := (\cdot)_{\text{R}} - (\cdot)_{\text{L}}$ of the primitive variables $\underline{v}_{\text{E}} := [\rho, u, v, w, p]^{\text{T}}$. By [26, Lemma 1, p. 633], the necessary and sufficient condition for a numerical flux to give the exact solution of the Riemann problem across a contact-discontinuity moving with speed U_n in the direction $\vec{e}_n$ ($V_{n_{\text{L}}} = V_{n_{\text{R}}}$, $p_{\text{L}} = p_{\text{R}}$, $\rho_{\text{L}} \neq \rho_{\text{R}}$), is $D_{\rho,\rho} = |U_n|$. Liou's condition [26, Lemma 1, p. 633] implies that the numerical massflux-dissipation at a stationary contact-discontinuity should be $\Delta_{\text{LR}}\rho$-independent.

In one of the earliest implementations of compressible RANS equations with RSM closure, Vandromme and Ha Minh [28] used the explicit-implicit MacCormack scheme [29], which is centred, in the sense that no preferential directions are identified with reference to the wave-structure of the Riemann problem [15], and $O(\Delta\ell^2)$. The mean-flow energy variable was the Favre-averaged total internal energy ($\tilde{e}_t := \tilde{e} + \frac{1}{2}\tilde{u}_i\tilde{u}_i + \text{k}$, where e is the internal energy, $\tilde{\cdot}$ represents Favre-averaging [30,31], $\text{k} := \frac{1}{2}\widetilde{u_i'' u_i''}$ is the turbulent kinetic energy associated with Favre fluctuations of the velocity-components). The so-called isotropic effective pressure $\bar{p} + \frac{2}{3}\bar{\rho}\text{k}$ was included [28] in the convective fluxes, while the anisotropic part of the Reynolds-stresses $\overline{\rho u_i'' u_j''} - \frac{2}{3}\bar{\rho}\text{k}\delta_{ij}$ appearing in the mean-flow momentum and energy equations, was included in the diffusive fluxes (centred discretization in both the predictor and corrector sweeps of MacCormack's scheme [29]). Vandromme and Ha Minh [28] included only the isotropic part of the Reynolds-stresses in the convective flux, because of difficulties, which have since been identified with the fact that the convective part of the RSM–RANS equations (without the nonconservative products [32] associated with Reynolds-stress production by mean-flow velocity-gradients, $P_{ij} := -\overline{\rho u_i'' u_\ell''}\partial_{x_\ell}\tilde{u}_j - \overline{\rho u_j'' u_\ell''}\partial_{x_\ell}\tilde{u}_i$) is not hyperbolic [33] because its Jacobian matrix does not have a complete system of eigenvectors [34–36]. Morrison [37] used an implicit $O(\Delta\ell^2)$ MUSCL [38] scheme with Roe fluxes [18], which are contact-discontinuity-resolving [26], with $\tilde{e}_t$ as mean-flow energy variable. The concept of isotropic effective pressure $\bar{p} + \frac{2}{3}\bar{\rho}\text{k}$ [28] was not used [37], and Reynolds-stresses in the mean-flow equations were simply included in the diffusive fluxes (centred discretization). Both these early studies [28,37] included computational examples of shock-wave/turbulent-boundary-layer interactions on structured grids. Chenault et al. [39] used Morrison's code [37] to compute the complex 3-D flow of a supersonic ejection in crossflow.

Traditionally, from a conceptual turbulence theory point-of-view, Reynolds-stresses are understood as an addition to viscous stresses accounting for the effects of turbulent mixing on the mean-flow [40, pp. 32–33]. Rautaheimo and Siikonen [34] were probably the first to recognize that, contrary to this conceptual description, Reynolds-stresses appear in the mean-flow equations as 1-derivatives, and should therefore be included in the convective fluxes and not in the viscous (diffusive) ones which regroup 2-derivatives. This is a fundamental mathematical difference with respect to 2-equation closures [41–45], whether linear [46–48] or nonlinear [49,50]. Within the framework of 2-equation closures, Reynolds-stresses are not variables of the system of PDEs (partial differential equations), but are instead replaced by a constitutive relation involving mean-flow velocity-gradients, and correctly appear in the diffusive fluxes of the mean-flow equations. Rautaheimo and Siikonen [34] also included the nonconservative products P_{ij} in the convective terms to obtain a (nonstrictly) hyperbolic system [51] and constructed Roe fluxes for this representative system [34,52,53]. Schwarz-inequality realizability constraints [54] were included in the eigenvector matrices [34, (3.22), p. 17] to avoid numerical instabilities. A simpler method, using the isotropic effective pressure concept, in line with Vandromme and Ha Minh [28], which treats P_{ij} as a