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A second-order accurate immersed boundary-lattice Boltzmann method for particle-laden flows

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ABSTRACT

A new immersed boundary-lattice Boltzmann method (IB-LBM) is presented for fully resolved simulations of incompressible viscous flows laden with rigid particles. The immersed boundary method (IBM) recently developed by Breugem (2012) [19] is adopted in the present method, development including the retraction technique, the multi-direct forcing method and the direct account of the inertia of the fluid contained within the particles. The present IB-LBM is, however, formulated with further improvement with the implementation of the high-order Runge–Kutta schemes in the coupled fluid–particle interaction. The major challenge to implement high-order Runge–Kutta schemes in the LBM is that the flow information such as density and velocity cannot be directly obtained at a fractional time step from the LBM since the LBM only provides the flow information at an integer time step. This challenge can be, however, overcome as given in the present IB-LBM by extrapolating the flow field around particles from the known flow field at the previous integer time step. The newly calculated fluid–particle interactions from the previous fractional time steps of the current integer time step are also accounted for in the extrapolation. The IB-LBM with high-order Runge–Kutta schemes developed in this study is validated by several benchmark applications. It is demonstrated, for the first time, that the IB-LBM has the capacity to resolve the translational and rotational motion of particles with the second-order accuracy. The optimal retraction distances for spheres and tubes that help the method achieve the second-order accuracy are found to be around 0.30 and −0.47 times of the lattice spacing, respectively. Simulations of the Stokes flow through a simple cubic lattice of rotational spheres indicate that the lift force produced by the Magnus effect can be very significant in view of the magnitude of the drag force when the practical rotating speed of the spheres is encountered. This finding may lead to more comprehensive studies of the effect of the particle rotation on fluid–solid drag laws. It is also demonstrated that, when the third-order or the fourth-order Runge–Kutta scheme is used, the numerical stability of the present IB-LBM is better than that of all methods in the literature, including the previous IB-LBMs and also the methods with the combination of the IBM and the traditional incompressible Navier–Stokes solver.

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1. Introduction

Particulate flows are involved in a large number of process systems. The direct numerical simulation (DNS) method has been widely used in simulating particulate flows. The conventional DNS methods, such as the finite volume (FVM) and finite element methods (FEM), are more desirable to simulate particulate flows with particles frozen in the computational domain. For particulate flows with freely-moving particles, the main obstacle with these methods comes from the frequent need of generating new, geometrically adapted grids at every advancing step, which is a time-consuming task especially for three-dimensional flows.

The lattice Boltzmann method (LBM), which originates from the lattice automata methods, is an efficient alternative to traditional techniques for solving the incompressible Navier–Stokes (NS) equations [1–4]. Ladd [5,6] successfully adopted the lattice Boltzmann method (LBM) for the DNS study of particulate flows. In his study, a fixed, Eulerian grid system is implemented to represent the flow field. The "bounce-back" rule [6] is applied to realize the no-slip condition on the solid–fluid interface. Therefore, the need of generating new adapted grids at every time step is eliminated. However, based on the "bounce-back" rule, the boundary of a particle is captured in a step-wise way, making the solid–fluid interface non-smooth. To circumvent this problem, the immersed boundary method (IBM) developed by Peskin [7] can be adopted. The immersed boundary method is a numerical scheme that dynamically simulates solid or elastic bodies immersed in a surrounding fluid. Its basic idea is to employ a fixed Cartesian grid for the discretization of the fluid phase and to resolve the solid–fluid interface by adding additional force terms to the governing equations.

Based on the immersed boundary approach proposed by Peskin [7], Goldstein et al. [8] used the so-called adaptive or feedback forcing scheme to introduce solid surfaces into the flow field governed by the Navier–Stokes equations. In their method, two free parameters are required to be chosen based on the flow conditions. Based on the work by Goldstein et al. [8], Derksen et al. [9] and Derksen and Van den Akker [10] formulated an adaptive force-field technique within the lattice-Boltzmann framework. Using the adaptive force-field technique with the LBM, Ten Cate et al. [11,12] simulated the sedimentation of a single sphere in an enclosure and also the movement of the colliding monodisperse spheres in a forced isotropic turbulence. For the computation of the force on the particles, the "calibrated diameter" obtained from prescribed simulation was used. An obvious drawback of using the "calibrated diameter" is that, as pointed out by Feng and Michaelides [13], by adjusting the calibrated diameter, the method also modifies all the other computational errors regardless of their origin. Due to this drawback, the solution obtained by their method is independent of grid resolution and may not converge as the mesh refines [11].

Feng and Michaelides [13] combined desired elements of the immersed boundary method, the direct forcing method [14] and the LBM. They added a forcing term in the momentum equation to enforce the no-slip condition on the boundary of a moving particle. The method was demonstrated to generate a smoother boundary for particles and to be capable of achieving higher Reynolds number flows. Using the direct forcing method, their method eliminates the need for the determination of the two relaxation parameters in the adaptive force field scheme used by Ten Cate et al. [11,12]. However, it is claimed that the IB-LBM only achieves the first-order accuracy due to the large IBM error. Uhlmann [15] also presented an improved method through the use of IBM in the traditional incompressible viscous flow solver. The main idea is to incorporate Peskin's regularized delta function approach [16] into a direct formulation of the fluid–solid interaction force so that the method allows for a smooth transfer between Eulerian and Lagrangian representations.

Recently, Kempe and Frohlich [17] proposed several enhancements of IBM which considerably improve accuracy and extend the range of applicability. An important step is a simple low-cost iterative procedure for the Euler–Lagrange coupling yielding a substantially better imposition of boundary conditions at the interface, even for large time steps. The procedure they adopted is indeed the multi-forcing method discussed by Luo et al. [18]. Furthermore, they designed an efficient integration step for the artificial flow field inside the particles, making the accessible ratios of particle density and fluid density down to 0.3 from around 1.0. Breugem [19] demonstrated that accuracy of the immersed boundary method could be increased to second order by adopting several new developed techniques. The method is based on the computationally efficient direct-forcing method adopted by Uhlmann [15]. Specifically, the IBM by Uhlmann was improved by a multi-direct forcing scheme, a slight retraction of the Lagrangian grid from the surface towards the interior of the particles with a fraction of the Eulerian grid spacing, and a new procedure to lower the accessible particle–fluid mass density ratios by a direct account of the inertia of the fluid contained within the particles. The numerical examples performed by Breugem [19] have shown that the retraction distance r_d has a strong influence on the effective particle diameter and little influence on the error in the no-slip/no-penetration (ns/np) condition, while exactly the opposite holds for the number of iterations N_s. The choice of $r_d/\Delta x = 0.3$ was found to yield second-order accuracy compared to first-order accuracy of the original method that corresponds to $r_d/\Delta x = 0$.

In both Kempe and Frohlich and Breugem's studies, the IBM is combined with the traditional incompressible solver, in which the incompressible Navier–Stokes equations are solved. It is known that, through the time discretization, the traditional NS solver can be easily executed at fractional steps using high-order time advancing techniques, such as Runge–Kutta schemes. However, Runge–Kutta schemes cannot be directly embedded in the LBM since the LBM only resolves the flow field at integer steps. This paper presents a novel approach that uses several typical Runge–Kutta schemes to combine the IBM proposed by Breugem and the LBM. At every inner stage of the Runge–Kutta schemes, the flow field around a particle is obtained by extrapolation based on the known flow field from the LBM at the previous integer time step. The known force terms representing the fluid–particle interactions from the previous inner time steps of the same integer time step are