Journal of Computational Physics 268 (2014) 456–484

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A robust and contact resolving Riemann solver on unstructured mesh, Part II, ALE method

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ARTICLE INFO

Article history:
Received 21 June 2013
Received in revised form 26 February 2014
Accepted 3 March 2014
Available online 18 March 2014

Keywords:
Godunov methods
Riemann solver
Moving mesh
ALE

ABSTRACT

This paper investigates solution behaviors under the strong shock interaction for moving mesh schemes based on the one-dimensional Godunov and HLLC Riemann solvers. When the grid motion velocity is close to Lagrangian one, these Godunov methods, which updates the flow parameters directly on the moving mesh without using interpolation, may suffer from numerical shock instability. In order to cure such instability, a new cell centered arbitrary Lagrangian Eulerian (ALE) algorithm is constructed for inviscid, compressible gas flows. The main feature of the algorithm is to introduce a nodal contact velocity and ensure the compatibility between edge fluxes and the nodal flow intrinsically. We establish a new two-dimensional Riemann solver based on the HLLC method (denoted by the ALE HLLC-2D). The solver relaxes the condition that the contact pressures must be the same in the traditional HLLC solver and constructs discontinuous fluxes along each sampling direction of the similarity solution. The two-dimensional contact velocity of the grid node is determined via enforcing conservation of mass, momentum and total energy. The resulting ALE scheme has a node instead of grid edge conservation properties. Numerical tests are presented to demonstrate the robustness and accuracy of this new solver. Due to the multi-dimensional information introduced and consistency between the fluxes and nodal contact velocity, the developed ALE algorithm performs well on both quadrilateral and triangular grids and reduces numerical shock instability phenomena.

1. Introduction

In many computational fluid dynamics (CFD) applications, boundaries of the physical domain of the flow might move in time. Typical examples include flows in airfoil oscillations, wing flutter, and a large class of multi-material interface problems where the material interface is explicitly tracked as a Lagrangian surface. When moving boundaries experience large displacements, or when they undergo large deformations, the arbitrary Lagrangian Eulerian method (ALE) is a useful tool to solve the flow problems on a moving and possibly deforming grid.

Usually, ALE methods can be implemented by three approaches. The first one, which is termed as Lagrange plus remap algorithm, is proposed by Hirt et al. [24]. It includes an explicit Lagrangian step, a rezoning step and a remapping step.

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http://dx.doi.org/10.1016/j.jcp.2014.03.003

Such a method is suitable for the problems in which the Lagrangian grid and rezoned grid are independent and even have different topologies. Recently such methods have been used in multi-material flows calculation with interface reconstruction or reconnection-based technique [21,22,33]. The second is Euler plus remap algorithm, which are used in extensive adaptive moving mesh literatures [14,31,32,47,48]. The last one is named direct ALE and consists in an unsplitting moving mesh discretization of the gas dynamics equations wherein grid topologies keep unchange and convective terms are solved directly, see [3,34,41].

The early study in this direction, as we know, can be found in [28] and [23], who proposed a method of moving mesh at an adaptive speed at each time step to improve the resolution of shocks and contact discontinuities. Then, many other moving mesh methods for hyperbolic problems have been studied in the literatures, and most of these works focus on grid moving strategies.

Due to the fact that the Lagrangian mesh reflects the physical motion of the fluid, the 'rezoned' grid is required to be close to the Lagrangian one at each time step in some grid rezone strategies used in ALE computation. Knupp, Margolin and Shashkov [29] construct a so-called 'reference Jacobian method' based on optimization to maintain both geometric and physical qualities of grid. Galera, Maire and Breil in [21] present a multi-material ALE scheme, whose main feature lies in introducing a new mesh relaxation procedure which also keeps the rezoned grid as close as possible to the Lagrangian one. Hui et al. construct 'the generalized Lagrangian method' in computational space [26] and introduce a unified coordinate system [27], where the main idea of the algorithm is to move grids with the same direction of the fluid movement but with different magnitude of fluid velocity.

Adaptive moving mesh is another well-known grid rezone strategy. Locally clustering mesh points in the regions where there are large gradients in the physical quantities will effectively reduce possible errors or oscillations. Up to now, much progress has been made in adaptive moving mesh methods for the numerical solution of partial differential equations, including grid redistribution approach based on the variational principle proposed by Azarenok et al. [3,4], Brackbill [10], Yanenko et al. [54] and Winslow [51]; moving mesh PDEs methods proposed by Huang [25]; and moving mesh methods based on the harmonic mapping proposed by Chen et al. [14], Dvinsky [19], Li et al. [31,32], and Tang et al. [47,48].

Some researchers have paid more attention to numerical schemes for flow problems. Zabrodin et al. [56] design a kind of ALE method on curvilinear grid. Azarenok et al. [3] discuss a second order ALE Godunov method on moving mesh. Farhat et al. [20] investigate the influence of geometric conservation law to nonlinear stabilities of ALE scheme. Luo, Baum and Lohner [34] implement AUSM+, HLLC, and Godunov schemes in the context of ALE formulation, and find that ALE AUSM+ scheme could lead to collapse of a calculation for a class of underwater explosion problems, whereas the ALE HLLC and Godunov schemes are able to offer accurate and robust solutions for capturing strong shock and contact discontinuities. Pakmor et al. [43] find that calculating an MHD problem turns out to be unstable when the grid velocities are close to the fluid velocity and then propose a modification technique.

This paper focuses on numerical method of moving grid and reports a numerical phenomenon: under the strong shock interaction, the Godunov and HLLC methods in ALE formulation in [34] also suffer from instability on some well-used moving grids. Such unstable phenomenon comes from a genuine two-dimensional mechanism since it depends on grid aspect ratio and shock strength. The modification method proposed by Parkmor et al. [43] may be effective in the computation of MHD problems, but it is invalid in gas flow for hydrodynamic problems.

Moreover, we present a new cell-centered ALE method to solve the two-dimensional compressible gas dynamics equations on unstructured grids. The main work is to construct a two-dimensional Riemann solver. Different from some classical multi-dimensional Riemann solvers such as those proposed by Balsara [5–7], Boscheri et al. [8,9] and Wendroff [52], which are based on the multi-dimensional wave configurations, our method provides a numerical flux at a cell face after given inputs to a Riemann problem. The method has close relation with a set of cell centered Lagrangian ones with nodal solvers.

The pioneers of the nodal solvers, e.g., Després and Mazeran [16], and Maire et al. [36], have noticed that the flux computation was not compatible with the node displacement in the early Lagrangian CAVEAT algorithm [17]. This incompatibility may generate additional spurious components in the vertex velocity field. In order to attain the consistency between the flux discretization and vertex velocity, they design an approximate Riemann solver located at the nodes and therefore modify the calculation results quite well. From then, a series of contributions to the development of the method are performed. Burton et al. extended the seminal works of [16,36] by proposing a new nodal Riemann-like method that handles solid dynamics [11], and Morgan et al. apply their nodal solver to contact surface algorithm [40]. Carré et al. extend Després and Mazeran's method to arbitrary dimension [13]. Maire develops high-order schemes on two-dimensional Cartesian and cylindrical geometries [37,38]. More work can be found in [15,33] and [39].

The present method in this paper is a generalization from the Eulerian method in [46] to the ALE method, which can be also seen as an extension of the nodal solvers in Lagrangian framework. The main work is to construct a two-dimensional Riemann solver whose flux has one-dimensional form but includes multi-dimensional information. The algorithm consists of two stages: (1) a two-dimensional contact velocity defined at grid vertex is determined by conservation of mass, momentum and total energy; (2) an approximate Riemann solver HLLC-2D is constructed in which the vertex velocity is given. The solver relaxes the condition that the contact pressures must be the same in the HLLC solver and constructs two discontinuous fluxes in any sampling direction of the similarity solution. The robustness and the accuracy of this new scheme are validated by numerical results.

The outline of this paper is as follows. In Section 2 the governing equations in the context of an ALE formulation and their space discretization are described. The ALE and Lagrangian formulations based on the one-dimensional Godunov and HLLC