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Non-Newtonian flow in porous media

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ABSTRACT

In this article we present a review of the single-phase flow of non-Newtonian fluids in porous media. The four main approaches for describing the flow through porous media in general are examined and assessed in this context. These are: continuum models, bundle of tubes models, numerical methods and pore-scale network modeling.

1. Introduction

Newtonian fluids are defined to be those fluids exhibiting a direct proportionality between stress τ and strain rate $\dot{\gamma}$ in laminar flow, that is

$$\tau = \mu\dot{\gamma} \tag{1}$$

where the viscosity μ is independent of the strain rate although it might be affected by other physical parameters, such as temperature and pressure, for a given fluid system [1,2]. All those fluids for which the proportionality between stress and strain rate is violated, due to nonlinearity or initial yield-stress, are said to be non-Newtonian. Some of the most characteristic features of non-Newtonian behavior are strain- and time-dependent viscosity, yield-stress, and stress relaxation. Non-Newtonian fluids are commonly divided into three broad groups: time-independent, viscoelastic and time-dependent. However, in reality these classifications are often by no means distinct or sharply defined [1,2]. Those fluids that exhibit a combination of properties from more than one of the above groups are described as complex fluids [3], though this term may be used for non-Newtonian fluids in general. A large number of rheological models have been proposed in the literature to model all types of non-Newtonian fluids under diverse flow conditions. However, the majority of these models are basically empirical in nature and arise from curve-fitting exercises [4]. A few prominent examples of the non-Newtonian models from the three groups are presented in Table 1.

1.1. Time-independent fluids

Time-independent fluids are those for which the strain rate at a given point is solely dependent upon the instantaneous stress at that point. Shear rate dependence is one of the most important and defining characteristics of non-Newtonian fluids in general and time-independent fluids in particular. When a typical non-Newtonian fluid experiences a shear flow the viscosity appears to be Newtonian at low shear rates. After this initial Newtonian plateau the viscosity is found to vary with increasing shear rate. The fluid is described as shear-thinning or pseudoplastic if the viscosity decreases, and shear-thickening or dilatant if the viscosity increases on increasing shear rate. After this shear-dependent regime, the viscosity reaches a limiting constant value at high shear rate. This region is described as the upper Newtonian plateau. If the fluid sustains initial stress without flowing, it is called a yield-stress fluid. Almost all polymer solutions that exhibit a shear rate dependent viscosity are shear-thinning, with relatively few polymer solutions demonstrating dilatant behavior. Moreover, in most known cases of shear-thickening there is a region of shear-thinning at lower shear rates [4–6].

Fig. 1 demonstrates the six principal rheological classes of the time-independent fluids in shear flow. These represent shear-thinning, shear-thickening and shear-independent fluids each with and without yield-stress. It is worth noting that these rheological classes are idealizations as the rheology of these fluids is generally more complex and they can behave differently under various deformation and ambient conditions. Prominent examples of the time-independent fluid models are: power-law, Ellis, Carreau and Herschel-Bulkley. These are widely used in modeling non-Newtonian fluids of this group [7].

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Table 1
Examples of non-Newtonian rheological models.

Model	Equation
Power-Law	$\mu = C\dot{\gamma}^{n-1}$
Ellis	$\mu = \frac{\mu_o}{1 + (\frac{\tau}{\tau_{1/2}})^{\alpha-1}}$
Carreau	$\mu = \mu_\infty + \frac{\mu_o - \mu_\infty}{[1 + (\dot{\gamma} t_c)^2]^{\frac{1-n}{2}}}$
Herschel-Bulkley	$\tau = \tau_o + C\dot{\gamma}^n (\tau > \tau_o)$
Maxwell	$\boldsymbol{\tau} + \lambda_1 \frac{\partial \boldsymbol{\tau}}{\partial t} = \mu_o \dot{\boldsymbol{\gamma}}$
Jeffreys	$\boldsymbol{\tau} + \lambda_1 \frac{\partial \boldsymbol{\tau}}{\partial t} = \mu_o (\dot{\boldsymbol{\gamma}} + \lambda_2 \frac{\partial \dot{\boldsymbol{\gamma}}}{\partial t})$
Upper Convected Maxwell	$\boldsymbol{\tau} + \lambda_1 \overset{\nabla}{\boldsymbol{\tau}} = \mu_o \dot{\boldsymbol{\gamma}}$
Oldroyd-B	$\boldsymbol{\tau} + \lambda_1 \overset{\nabla}{\boldsymbol{\tau}} = \mu_o (\dot{\boldsymbol{\gamma}} + \lambda_2 \overset{\nabla}{\dot{\boldsymbol{\gamma}}})$
Godfrey	$\mu(t) = \mu_i - \Delta\mu'(1 - e^{-t/\lambda'}) - \Delta\mu''(1 - e^{-t/\lambda''})$
Stretched Exponential Model	$\mu(t) = \mu_i + (\mu_{\inf} - \mu_i)(1 - e^{-(t/\lambda_s)^c})$

∇ upper convected time derivative, α rheological parameter, $\dot{\gamma}$ rate of strain, $\dot{\boldsymbol{\gamma}}$ rate of strain tensor, λ_1 relaxation time, λ_2 retardation time, λ' λ'' λ_s time constants, μ viscosity, μ_i initial-time viscosity, $\mu_{\inf}$ infinite-time viscosity, μ_o zero-shear viscosity, μ_∞ infinite-shear viscosity, $\Delta\mu'$ $\Delta\mu''$ viscosity deficits, τ stress, $\boldsymbol{\tau}$ stress tensor, $\tau_{1/2}$ stress when $\mu = \mu_o/2$, τ_o yield-stress, c dimensionless constant, C consistency factor, n flow behavior index, t time, t_c characteristic time of flow system.

1.2. Viscoelastic fluids

Viscoelastic fluids are those that show partial elastic recovery upon the removal of a deforming stress. Such materials possess properties of both viscous fluids and elastic solids. Polymeric fluids often show strong viscoelastic effects. These include shear-thinning, extension-thickening, normal stresses, and time-dependent rheology. No theory is yet available that can adequately describe all of the observed viscoelastic phenomena in a variety of flows. Nonetheless, many differential and integral constitutive models have been proposed in the literature to describe viscoelastic flow. What is common to all these is the presence of at least one characteristic time parameter to account for the fluid memory, that is the stress at the present time depends upon the strain or rate of strain for all past times, but with a fading memory [6,8–11].

Broadly speaking, viscoelasticity is divided into two major fields: linear and nonlinear. Linear viscoelasticity is the field of rheology devoted to the study of viscoelastic materials under very small strain or deformation where the displacement gradients are very small and the flow regime can be described by a linear relationship between stress and rate of strain. In principle, the strain has to be sufficiently small so that the structure of the material remains unperturbed by the flow history. If the strain rate is small enough, deviation from linear viscoelasticity may not occur at all. The equations of linear viscoelasticity are not valid for deformations of arbitrary magnitude and rate because they violate the principle of frame invariance. The validity of linear viscoelasticity when the small-deformation condition is satisfied with a large magnitude of rate of strain is still an open question, though it is widely accepted that linear viscoelastic constitutive equations are valid in general for any strain rate as long as the total strain remains small. Nevertheless, the higher the strain rate the shorter the time at which the critical strain for departure from linear regime is reached [5,12,13].

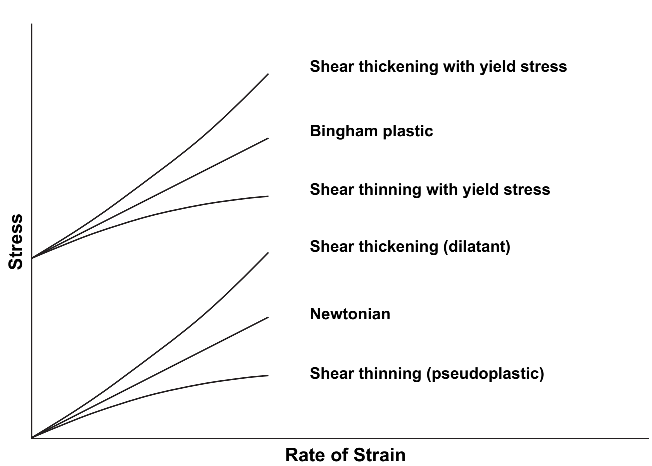

Fig. 1. The six main classes of the time-independent fluids presented in a generic graph of stress against strain rate in shear flow.

The linear viscoelastic models have several limitations. For example, they cannot describe strain rate dependence of viscosity or normal stress phenomena since these are nonlinear effects. Due to the restriction to infinitesimal deformations, the linear models may be more appropriate for the description of viscoelastic solids rather than viscoelastic fluids. Despite the limitations of the linear viscoelastic models and despite being of less interest to the study of flow where the material is usually subject to large deformation, they are very important in the study of viscoelasticity for several reasons [5,12,14]:

- They are used to characterize the behavior of viscoelastic materials at small deformations.
- They serve as a motivation and starting point for developing nonlinear models since the latter are generally extensions to the linears.
- They are used for analyzing experimental data obtained in small deformation experiments and for interpreting important viscoelastic phenomena, at least qualitatively.

The two most prominent linear viscoelastic fluid models are the Maxwell and Jeffreys.

Nonlinear viscoelasticity is the field of rheology devoted to the study of viscoelastic materials under large deformation, and hence it is the subject of primary interest to the study of flow of viscoelastic fluids. Nonlinear viscoelastic constitutive equations are sufficiently complex that very few flow problems can be solved analytically. Moreover, there appears to be no differential or integral constitutive equation general enough to explain the observed behavior of polymeric systems undergoing large deformations but still simple enough to provide a basis for practical applications [1,5,15].

As the linear viscoelasticity models are not valid for deformations of large magnitude because they do not satisfy the principle of frame invariance, Oldroyd and others tried to extend these models to nonlinear regimes by developing a set of frame-invariant constitutive equations. These equations define time derivatives in frames that deform with the material elements. Examples include rotational, upper and lower convected time derivatives. The idea of these derivatives is to express the constitutive equation in real space coordinates rather than local coordinates and hence fulfilling the Oldroyd's admissibility criteria for constitutive equations. These admissibility criteria ensure that the equations are invariant under a change of coordinate system, value invariant under a change of translational or rotational motion of the fluid element as it goes through space, and value invariant under a change of rheological history of neighboring fluid elements [5,14].

There is a large number of rheological equations proposed for the description of nonlinear viscoelasticity, as a quick survey to the literature reveals. However, many of these models are extensions or modifications to others. The two most popular nonlinear viscoelastic models in differential form are the Upper Convected Maxwell and the Oldroyd-B models. Figs. 2–4 display several aspects of the rheology of viscoelastic fluids in bulk and *in situ*. In Fig. 2 a stress versus time graph reveals a distinctive feature of time dependency largely observed in viscoelastic fluids. As seen, the