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A nonlinear modeling approach using weighted piecewise series and its applications to predict unsteady flows

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ABSTRACT

To preserve nonlinearity of a full-order system over a range of parameters of interest, we propose an accurate and robust nonlinear modeling approach by assembling a set of piecewise linear local solutions expanded about some sampling states. The work by Rewienski and White [1] on micromachined devices inspired our use of piecewise linear local solutions to study nonlinear unsteady aerodynamics. These local approximations are assembled via nonlinear weights of radial basis functions. The efficacy of the proposed procedure is validated for a two-dimensional airfoil moving with different pitching motions, specifically AGARD's CT2 and CT5 problems [27], in which the flows exhibit different nonlinear behaviors. Furthermore, application of the developed aerodynamic model to a two-dimensional aero-elastic system proves the approach is capable of predicting limit cycle oscillations (LCOs) by using AGARD's CT6 [28] as a benchmark test. All results, based on inviscid solutions, confirm that our nonlinear model is stable and accurate, against the full model solutions and measurements, and for predicting not only aerodynamic forces but also detailed flowfields. Moreover, the model is robust for inputs that considerably depart from the base trajectory in form and magnitude. This modeling provides a very efficient way for predicting unsteady flowfields with varying parameters because it needs only a tiny fraction of the cost of a full-order modeling for each new condition—the more cases studied, the more savings rendered. Hence, the present approach is especially useful for parametric studies, such as in the case of design optimization and exploration of flow phenomena.

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1. Introduction

Most aerodynamics problems of interest are highly nonlinear; they are being routinely studied with validated computational fluid dynamics (CFD) codes, thanks to the availability of fast-improving computer technologies. The need for reducing the computation time, however, is becoming ever stronger because more details (thus more grid points and higher fidelity) are demanded. Further, additional utilization of CFD is increasingly realized for other applications, such as aerodynamic analysis and design optimization by varying a range of input parameters, and the coupling of aerodynamics with other dis-

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Nomenclature

b, c	semi-chord and chord, respectively	I_α	second moment of inertia of airfoil about elastic axis
M_∞	freestream Mach number	x_α	airfoil static unbalance
α, h	pitch and plunge displacement	q_∞	freestream dynamic pressure
α_m, α_0	mean angle of attack and pitch oscillation amplitude	C_L, C_M	lift and moment coefficient, respectively
T, dt	period and time step	ω_h, ω_α	pitch and plunge natural frequency
m	wing mass	ρ_∞	freestream density
S_α	first moment of inertia of airfoil about elastic axis	U_∞	freestream velocity
K_h, K_α	plunge stiffness and torsional stiffness about elastic axis	V	reduced freestream speed
		μ	mass ratio

ciplines (like structure, control, etc.). For design optimization, repeated computations of similar setup are performed with design variables varying in values over the design space, thus rendering the computation costly. Thus, there is a strong need to reduce the overall computational effort, either through a speed up by employing advances in computer technology or by devising an efficient numerical formulation. One idea is to replace the original full model with an approximate model that will require minimal additional computational efforts. This may be achieved in two ways. One, under the name of reduced order method or model order reduction, aims to find an approximation with the number of unknowns much smaller than the full system; and the other, under the names such as surrogate modeling, metamodeling, and so forth, bases on a set of available data (empirical or computational fluid dynamics (CFD) solutions) to build a model that is valid inside the parameters (state) range. Both ideas will be further explored later in this section. The key criterion for both is that the fidelity of the original model must be preserved for a sufficiently large range so that it is useful in practice. An important aspect of this criterion is the ability to preserve nonlinear characteristics of aerodynamics. This is the goal of the research reported here: robust modeling for accuracy with efficiency.

Our research on the subject of the present paper has been motivated by the need to perform multidisciplinary analysis and optimization for an aeronautical system under NASA's Fundamental Aeronautics Program. In particular, aeroelasticity is the subject of interest—we are concerned with the coupling of unsteady aerodynamics and structure dynamics and its interactive effects, especially the limit cycle oscillations, for which numerous input parameters must be varied.

Conventional aeroelastic stability analysis is often formulated simply in terms of an input–output relationship, where input is aerodynamic force and output is structural response. The system identification (ID) method [2] is a straightforward framework to construct a simple reduced order model (ROM), but at the expense of detailed flowfield information. However, such information about flowfield characteristics will be required, for example, for finding a new aerodynamic shape to minimize occurrence of structure flutter in an aeroelastic optimization problem.

To retain flowfield details, the state-space projection method based on modes superposition [4] has often been employed. Unfortunately, the linear superposition of modes limits the method from being able to capture nonlinear phenomena such as shock wave dynamics or limit cycle oscillations [7].

The basic concept of the state-space projection method is to find a subspace onto which the full-order system can be projected with a much smaller dimension. Proper orthogonal decomposition (POD) is a well-known method to define such subspace. It is a method primarily applied to unsteady flow problem with intent of accurately and efficiently representing the flowfield; the main idea is to construct a new system with reduced degrees of freedom. Sirovich [3] used the method of snapshot to study the turbulence coherent structures. Romanowski [4] applied POD or Karhunen–Loéve expansion to build a reduced order model of an isolated airfoil aero-elastic system. A key step in POD is to collect snapshots of the time-dependent solution. Once a snapshot matrix is formed, the subspace can be derived from eigenvalue analysis. Traditionally, POD merely takes input characteristics into account. Moore [5] introduced the balance truncation (BT) method to define the subspace by including both the input and output characteristics of the full-order system. As a result, the BT method often yields a more accurate and lower-order ROM than POD alone. Laub [6] proposed an efficient way to derive the transformation vector. Willcox and Peraire [7] and Rowley [8] have each applied the concept of BT method together with snapshot ideas (or BPOD) to study fluid dynamics problems. Ma et al. [9] reveals that eigensystem realization algorithm can construct an equivalent ROM as BPOD, however, but doesn't require adjoint or dual systems; it has recently been successfully used with experimental data [10].

As stated, our primary interest is to focus on capturing nonlinearity in a flow or aeroelastic system; in what follows we shall present some existing relevant methods that preserve of the full-order model. It is relatively straightforward to obtain a *linear* model of a highly nonlinear system by retaining only the first-order terms; this can be done, for example, by finite differencing a CFD code or using automatic differentiation tools [11]. However, it remains challenging to obtain a *nonlinear* model of the CFD code, as elaborated below. Let us consider a time-dependent nonlinear system,

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \tag{1}$$