Journal of Computational Physics 318 (2016) 277–304

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An algorithm for computing the 2D structure of fast rotating stars

Michel Rieutord [a,b,*], Francisco Espinosa Lara [a,b,1], Bertrand Putigny [a,b]

[a] Université de Toulouse, UPS-OMP, IRAP, Toulouse, France
[b] CNRS, IRAP, 14, avenue Edouard Belin, F-31400 Toulouse, France

ARTICLE INFO

Article history:
Received 11 February 2016
Received in revised form 2 May 2016
Accepted 3 May 2016
Available online 9 May 2016

Keywords:
Astrophysics
Stellar models

ABSTRACT

Stars may be understood as self-gravitating masses of a compressible fluid whose radiative cooling is compensated by nuclear reactions or gravitational contraction. The understanding of their time evolution requires the use of detailed models that account for a complex microphysics including that of opacities, equation of state and nuclear reactions. The present stellar models are essentially one-dimensional, namely spherically symmetric. However, the interpretation of recent data like the surface abundances of elements or the distribution of internal rotation have reached the limits of validity of one-dimensional models because of their very simplified representation of large-scale fluid flows. In this article, we describe the ESTER code, which is the first code able to compute in a consistent way a two-dimensional model of a fast rotating star including its large-scale flows. Compared to classical 1D stellar evolution codes, many numerical innovations have been introduced to deal with this complex problem. First, the spectral discretization based on spherical harmonics and Chebyshev polynomials is used to represent the 2D axisymmetric fields. A nonlinear mapping maps the spheroidal star and allows a smooth spectral representation of the fields. The properties of Picard and Newton iterations for solving the nonlinear partial differential equations of the problem are discussed. It turns out that the Picard scheme is efficient on the computation of the simple polytropic stars, but Newton algorithm is unsurpassed when stellar models include complex microphysics. Finally, we discuss the numerical efficiency of our solver of Newton iterations. This linear solver combines the iterative Conjugate Gradient Squared algorithm together with an LU-factorization serving as a preconditioner of the Jacobian matrix.

1. Introduction

The recent progress of observational stellar astrophysics in spectroscopy, spectropolarimetry or interferometry have called for more realistic models of stars, with a focus on the effects of rotation. Without rotation stars may be modeled as spherical ‘balls’ with a detailed microphysics: equation of state, opacities or nuclear reaction rates have been the subject of intense research over the past fifty years [32,28]. In these one-dimensional models, the main difficulty comes from the modeling of the averaged heat transport by convection in the various parts of the star where hydrostatic equilibrium is unstable. For stars burning hydrogen on the so-called main sequence, these regions are a convective core when the stellar mass is larger

* Corresponding author.
E-mail address: mrieutord@irap.omp.eu (M. Rieutord).
[1] Present address: Space Research Group, University of Alcalá, 28871 Alcalá de Henares, Spain.

http://dx.doi.org/10.1016/j.jcp.2016.05.011
0021-9991/

than 1.3 solar mass – hereafter noted $M_\odot$ – and a convective envelope when the mass is less than 1.8 $M_\odot$. One-dimensional models have been designed and redesigned for more than fifty years now and are still widely used (e.g. the code MESA started by Paxton et al. [39]). They have had great successes in depicting a now widely accepted view of stellar evolution.

But, as alluded above, the more precise observations obtained with modern instruments show details that are difficult to explain with one-dimensional, spherically symmetric models. Most of these details are related to fluid flows in the stars. We easily understand that it is uneasy to model fluid flows in one dimension. The bulk effects of rotation are the first victims of an imposed spherical symmetry. Current one-dimensional codes, like MESA or CESAM [34], include a modeling of rotation through its average effects: these are mainly the centrifugal effect and radial differential rotation that mimic baroclinic flows. As expected, these models show discrepancies when compared to observational data. For instance, they have difficulties to reproduce the abundances of elements at the surface of stars [4] or they simply cannot be used to interpret interferometric observations of fast rotating stars [33].

To overcome these difficulties, the natural step forward is to relax the spherical symmetry in the modeling and to work with models owing two dimensions of space at least. Thus, fluid flows can be computed more realistically and rotational effects as well. The first step in this direction is to elaborate two-dimensional axisymmetric models of stars. The centrifugal distortion of the star can then be naturally included as well as the global steady flows.

Attempts to build such models have begun in the sixties [26], almost at the same time as 1D models. First steps in the quest of 2D stellar models for fast rotating stars have been marked by a series of works starting with the one of Ostriker and Mark [37] who introduced the Self-Consistent Field (SCF) method[2] (see [41, for a short historical review]). In a subsequent series of papers, Clement [7–10] proposed another way of solving Poisson's equation by using finite differences, while later on Eriguchi and Müller [15,16] introduced a linear mapping $r_i(\theta_k) = \zeta_i R_s(\theta_k)$ such that the grid $r_i(\theta_k)$ automatically adjusts to the shape of the star (here given by its colatitude dependent radius $R(\theta)$). More recently, Roxburgh [46,47] reconsidered 2D models of fast rotating stars for asteroseismic purposes, while Jackson et al. [24,25] reconsidered similar models for interpreting the very flattened shape of the Be star Achernar, as revealed by the first precise interferometric observations of this star [13]. At the same time, Jackson et al. [24,25] improved the SCF method. Recent results of MacGregor et al. [31] presented SCF models with very high angular momentum showing stellar models with very strongly distorted shapes compared to the sphere. In an other line of research, Deupree [11] also computed 2D models that he later used to interpret recent interferometric and asteroseismic data obtained for the nearby fast rotating star Rasalhague [12]. However, in all the foregoing work the internal rotation of the star had to be specified (either as a solid body rotation or as a given differential rotation). In real isolated stars, differential rotation emerges from the baroclinic torque and Reynolds stresses, the former being prominent in radiative zones and the latter in convective regions. The first models that included self-consistently the pressure, density, temperature distributions and the associated baroclinic torque have been presented in Espinosa Lara and Rieutord [17] and later, using the proper spheroidal geometry, in Rieutord and Espinosa Lara [43], Espinosa Lara and Rieutord [18].

The main difficulty was to find the appropriate algorithm that allowed convergence of the iterations to the quasi-steady state of a fast rotating star consistently with the mean flows that pervade the whole star. Indeed, these flows face extremely large density variations (typically eight orders of magnitude) making solutions prone to numerical instabilities. In addition, heat transfer depends on the strongly varying heat conductivity (controlled by the fluid opacity) or on a vigorous turbulent convection. Even if thermal convection is modeled by a smooth mean-field approach, the global rapid variations of transport coefficients, especially near the surface, make the problem thorny.

The aim of this paper is to present to the readers the numerical side of the solution that we have found to the modeling of fast rotating main sequence stars as illustrated in Fig. 1 and 2. This solution is now used in the ESTER code, which is freely available at http://ester-project.github.io/ester/. A detailed discussion of the physical and astrophysical hypothesis of the ESTER models may be found in Espinosa Lara and Rieutord [18] or Rieutord and Espinosa Lara [44]. In the following, we shall first present the set of equations to be solved (sect. 2) and continue on presenting the mapping that is used to deal with the spheroidal shape of the star (sect. 3). We then introduce our choice of the discretization (spectral methods) in section 4 and discuss the choice of the algorithm (sect. 5). We finally illustrate the results with examples showing the numerical efficiency of the ESTER code at computing various stellar models (sect. 6). Conclusions and outlooks end the paper.

2. Mathematical formulation

2.1. Equations of stellar structure

Basically equations that are governing the structure of stars are those governing a compressible self-gravitating fluid flow with heat sources. Because of the very high temperatures of the central regions, heat sources are coming from nuclear

[2] Briefly, this method use's the formal solution of Poisson's equation in term of the density distribution, i.e.

$$\phi(\mathbf{x}) = -G\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3\mathbf{x}'$$

which has the great advantage of including the boundary conditions on ϕ at infinity. The potential is used to find a new ρ itself leading to a new potential.