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A third-order compact gas-kinetic scheme on unstructured meshes for compressible Navier–Stokes solutions

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ABSTRACT

In this paper, for the first time a third-order compact gas-kinetic scheme is proposed on unstructured meshes for the compressible viscous flow computations. The possibility to design such a third-order compact scheme is due to the high-order gas evolution model, where a time-dependent gas distribution function at cell interface not only provides the fluxes across a cell interface, but also presents a time accurate solution for flow variables at cell interface. As a result, both cell averaged and cell interface flow variables can be used for the initial data reconstruction at the beginning of next time step. A weighted least-square procedure has been used for the initial reconstruction. Therefore, a compact third-order gas-kinetic scheme with the involvement of neighboring cells only can be developed on unstructured meshes. In comparison with other conventional high-order schemes, the current method avoids the Gaussian point integration for numerical fluxes along a cell interface and the multi-stage Runge–Kutta method for temporal accuracy. The third-order compact scheme is numerically stable under CFL condition CFL $\approx$ 0.5. Due to its multidimensional gas-kinetic formulation and the coupling of inviscid and viscous terms, even with unstructured meshes, the boundary layer solution and vortex structure can be accurately captured by the current scheme. At the same time, the compact scheme can capture strong shocks as well.

1. Introduction

In computational fluid dynamics, the second-order methods are generally robust and reliable, and they are routinely employed in the practical computations. For the same computational cost, the higher-order methods can theoretically provide more accurate solutions, but they are less robust and more complicated. In recent decades, there have been continuous interests and efforts on the development of higher-order schemes. For engineering applications, the construction of higher-order numerical schemes on unstructured meshes becomes extremely demanding. There are a gigantic amount of publications about the introduction and survey of higher-order schemes. The current paper will mainly concentrate on the construction of third-order compact gas-kinetic scheme on unstructured meshes.

The gas-kinetic scheme (GKS) has been developed systematically for the compressible flow computations [25,26,14,10]. An evolution process from kinetic scale to hydrodynamic scale has been constructed for the flux evaluation. The kinetic

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effect through particle free transport contributes to the capturing of shock waves, and the hydrodynamic effect plays a dominant role for the capturing of resolved viscous and heat conducting solutions. In other words, the highly non-equilibrium gas distribution function in discontinuous region provides a physically consistent mechanism in the construction of numerical shock structure. In this sense, the gas-kinetic scheme is close to the upwind approach with implicit artificial dissipation, but with a different dissipative mechanism. In smooth flow region, the hydrodynamic scale physics corresponding to the multi-dimensional central difference discretization is automatically used for the capturing of viscous solutions. Due to the coupling of inviscid and viscous terms in the kinetic formulation, theoretically there is no fundamental barrier for the finite volume gas-kinetic scheme to capture Navier–Stokes solutions with structured or unstructured meshes. With the discretization in particle velocity space, a unified gas-kinetic scheme has been developed for the transport process in the entire flow regime from rarefied to continuum ones [27,18,9,23].

Recently, with the incorporation of higher-order initial data reconstruction, a higher-order gas-kinetic scheme has been proposed in [16,15,17]. The flux evaluation is based on the time evolution of flow variables from initial piece-wise discontinuous polynomials (parabolas) around a cell interface, where the spatial and temporal derivatives of gas distribution function are coupled nonlinearly. The distributions of flow variables around a cell interface interact through particle transport and collision in the determination of flux function. Besides the evaluation of a time-dependent flux function across a cell interface, the third-order gas evolution model provides an accurate time-dependent cell interface solution for flow variables as well. Thus, it is feasible to develop a compact scheme with the consideration of both cell averaged and point-wise cell interface values. A compact third-order gas-kinetic scheme has been proposed recently for the compressible Euler and Navier–Stokes equations on structured meshes with WENO-type reconstruction [21]. However, this reconstruction technique is difficult to be extended to unstructured meshes. Therefore, in this paper, a weighted least-square reconstruction will be used on unstructured meshes. To the third-order accuracy, a quadratic distribution for the flow variables inside each cell needs to be determined. Based on the cell averaged and cell interface values from neighboring cells only, an over-determined linear system is formed. With the weighted least-square solution for this system, the whole flow distribution can be fully determined. The shock detector can be used as well to switch between higher-order (3rd) and lower order (2nd) reconstructions in different regions. In comparison with traditional schemes, the Gaussian point integration for flux evaluation along a cell interface and the multi-stage Runge–Kutta method for temporal accuracy are avoided in the current compact scheme. At the same time, the third-order compact scheme is stable under the CFL condition CFL ≈ 0.5.

This paper is organized as follows. In Section 2, the finite volume scheme on unstructured meshes and third-order gas-kinetic flux solver are introduced. In Section 3, the compact reconstruction on unstructured meshes is presented. Section 4 includes numerical examples to validate the current algorithm. The last section is the conclusion.

2. Finite volume gas-kinetic scheme

2.1. Finite volume scheme on unstructured meshes

The two-dimensional gas-kinetic BGK equation [3] can be written as

$$f_t + \mathbf{u} \cdot \nabla f = \frac{g - f}{\tau}, \tag{1}$$

where f is the gas distribution function, g is the corresponding equilibrium state, and τ is the collision time. The collision term satisfies the following compatibility condition

$$\int \frac{g - f}{\tau} \varphi d\Xi = 0, \tag{2}$$

where $\varphi = (1, u, v, \frac{1}{2}(u^2 + v^2 + \xi^2))$, $d\Xi = dudvd\xi_1 \dots d\xi_K$, K is the number of internal degree of freedom, i.e. $K = (4 - 2\gamma)/(\gamma - 1)$ for two-dimensional flows, and γ is the specific heat ratio.

Based on the Chapman–Enskog expansion of BGK model, the Euler and Navier–Stokes, Burnett, and super-Burnett equations can be derived [4,19,25]. In the smooth region, the gas distribution function can be expanded as

$$f = g - \tau D_{\mathbf{u}} g + \tau D_{\mathbf{u}}(\tau D_{\mathbf{u}}) g - \tau D_{\mathbf{u}}[\tau D_{\mathbf{u}}(\tau D_{\mathbf{u}}) g] + \dots,$$

where $D_{\mathbf{u}} = \partial/\partial t + \mathbf{u} \cdot \nabla$. By truncating on different orders of τ, the corresponding macroscopic equations can be derived. For the Euler equations, the zeroth order truncation is taken, i.e. $f = g$. For the Navier–Stokes equations, the first order truncation is

$$f = g - \tau(u g_x + v g_y + g_t). \tag{3}$$

Based on the higher order truncations, the Burnett and super-Burnett equations can be also derived.

In this section, the control volumes are simply triangles. For each control volume Ω_i, its boundary is given by three line segments