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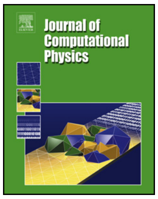

A stabilised nodal spectral element method for fully nonlinear water waves

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ABSTRACT

We present an arbitrary-order spectral element method for general-purpose simulation of non-overturning water waves, described by fully nonlinear potential theory. The method can be viewed as a high-order extension of the classical finite element method proposed by Cai et al. (1998) [5], although the numerical implementation differs greatly. Features of the proposed spectral element method include: nodal Lagrange basis functions, a general quadrature-free approach and gradient recovery using global L^2 projections. The quartic nonlinear terms present in the Zakharov form of the free surface conditions can cause severe aliasing problems and consequently numerical instability for marginally resolved or very steep waves. We show how the scheme can be stabilised through a combination of over-integration of the Galerkin projections and a mild spectral filtering on a per element basis. This effectively removes any aliasing driven instabilities while retaining the high-order accuracy of the numerical scheme. The additional computational cost of the over-integration is found insignificant compared to the cost of solving the Laplace problem. The model is applied to several benchmark cases in two dimensions. The results confirm the high order accuracy of the model (exponential convergence), and demonstrate the potential for accuracy and speedup. The results of numerical experiments are in excellent agreement with both analytical and experimental results for strongly nonlinear and irregular dispersive wave propagation. The benefit of using a high-order – possibly adapted – spatial discretisation for accurate water wave propagation over long times and distances is particularly attractive for marine hydrodynamics applications.

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1. Introduction

Robust and cost-efficient time-dependent simulation of the propagation and transformation of water waves in both shallow near-shore and deeper off-shore areas is a computationally challenging and longstanding scientific problem for ocean, coastal and naval engineering applications. For example, fully non-linear wave simulations have been subject to research for a long time, and have still not yet entered common coastal and ocean engineering practice. One remaining key challenge is to resolve accurately highly nonlinear and dispersive wave propagation in maritime areas while taking into account varying bathymetry, the geometry of complex structures and their nonlinear interaction with fixed and floating

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structures. Resolving this problem leads to improved opportunities for using simulations in realistic marine regions as well as enabling experiments in numerical wave tanks of increasing fidelity. Furthermore, it is attractive to design a flexible computational framework that entails use on modern commodity workstations as well as high-performance computing systems. These goals dictate stringent requirements on the design of engineering tools, and this suggests that a generic tool for wave propagation should be based on

(i) a general modelling basis for broadly describing relevant *physics*,
(ii) a generalised computational framework for the *numerics*, and
(iii) software design for efficient mapping to modern and emerging many-core *architectures*.

Our main objective is to meet each of these requirements via a spectral element based computational framework. Such a framework has already been used for several different applications within marine hydrodynamics such as the fully nonlinear potential flow equations (present work), Boussinesq and shallow water equations [20]. In this work we focus exclusively on the first two requirements and we pave the way for the fulfilment of the last one, which is being addressed in ongoing work. The use of modern many-core hardware is attractive for acceleration of the high-order spectral element framework and for enabling practical computations of realistic engineering problems [49,50].

1.1. Choice of modelling basis for description of physics

During the last decades computationally efficient depth-integrated Boussinesq-type models have been widely adopted as essential tools for water wave modelling in the near-shore region; see e.g. [48,4]. For shorter waves, such as the ones arising in offshore and naval engineering, Boussinesq-type models are not applicable due to the limited accuracy in terms of dispersive and nonlinear properties. For these cases we have to turn to Computational Fluid Dynamics (CFD) models based on the Navier–Stokes equations, or fully nonlinear potential flow (FNPF) models. The CFD models take viscous effects into account; effects that may be important for breaking waves, load computations, boundary layer effects, etc. Even though CFD is often prohibitively expensive in terms of computational resources when considering simulation of entire sea states [22], it is widely used to quantify breaking wave loads and wave run-up on structures. CFD models are typically too dissipative as a result of the low-order accuracy imposed by computational limitations for large-scale wave simulations. In contrast, already today FNPF models can be used for long-time and large-scale wave simulations [12,23]. FNPF solvers can be used for resolution of full sea states in large marine or coastal areas where nonlinear waves interact with fixed or floating structures. The cons of the FNPF models are that they cannot account for non-overturning breaking waves and viscous effects. For these reasons it can be attractive to combine FNPF models (far-field) with CFD (near-field) in hybrid modelling approaches for wave structure interaction, cf. [55]. This hybrid approach enables better simulation of strong nonlinear wave structure interactions in areas where the local wave climates can not be predicted accurately via a FNPF model.

1.2. On the quest towards developing numerical strategies for real-world applications

A review of existing conventional discretisation methods and applications reveals that historically the main emphasis has been on Finite Difference Methods (FDM), boundary element methods (BEM) and finite element methods (FEM) [37,29]. These methods have been designed for the concept of Numerical Wave Tanks (NWT). The main computational bottleneck in all such numerical solvers is the solution of a large linear system. In FDM and FEM the discretisation procedure leads to sparse linear systems due to the local support of discrete operators, while in BEM it is only the domain boundary that needs representation. The discretisation procedure for BEM is based on a surface integral formulation together with Green's identities. This leads to dense non-symmetric matrix operators that cannot be solved in a straightforward way with linear asymptotic scaling. There has been some recent progress in bringing the asymptotic cost (scaling rate) down for BEM [29] for both matrix–vector multiplications and storage requirements using both high-order basis functions and the Fast Multipole Method (FMM) [27]. While this strategy can asymptotically achieve linear complexity $\mathcal{O}(n)$ (n number of computational nodes) in work effort for the spatial solver, it has a large constant in front of this asymptotic scaling term due to the need of solving a dense linear system of equations. This leaves BEM less efficient compared to volume-based discretisation methods such as FDM and FEM solvers as suggested in [72,57]. We note, that BEM is particularly attractive as a near-field solver for cases where waves interact with complex geometries [75] and may be combined with a far-field solver such as FEM [73]. The overall efficiency and scalability of BEM [28] can be compared to efficient and massively parallel free surface hydrodynamics solvers such as [19,16,57] which can achieve very high efficiency and scalability using multigrid-type methods [42,14] for arbitrary sized discrete problems, in particular when the (possibly curvilinear multi-block) meshes are logically structured, e.g. as in [23].

1.3. State-of-the-art in finite element methods for fully nonlinear water waves

Reviews on the state-of-the art of numerical models for freely propagating water waves are given in [37,62,11,59,53]. Our scope in the present work is restricted to FNPF solvers and FEM.

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