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A stable partitioned FSI algorithm for incompressible flow and deforming beams

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ABSTRACT

An added-mass partitioned (AMP) algorithm is described for solving fluid–structure interaction (FSI) problems coupling incompressible flows with thin elastic structures undergoing finite deformations. The new AMP scheme is fully second-order accurate and stable, without sub-time-step iterations, even for very light structures when added-mass effects are strong. The fluid, governed by the incompressible Navier–Stokes equations, is solved in velocity-pressure form using a fractional-step method; large deformations are treated with a mixed Eulerian-Lagrangian approach on deforming composite grids. The motion of the thin structure is governed by a generalized Euler–Bernoulli beam model, and these equations are solved in a Lagrangian frame using two approaches, one based on finite differences and the other on finite elements. The key AMP interface condition is a generalized Robin (mixed) condition on the fluid pressure. This condition, which is derived at a continuous level, has no adjustable parameters and is applied at the discrete level to couple the partitioned domain solvers. Special treatment of the AMP condition is required to couple the finite-element beam solver with the finite-difference-based fluid solver, and two coupling approaches are described. A normal-mode stability analysis is performed for a linearized model problem involving a beam separating two fluid domains, and it is shown that the AMP scheme is stable independent of the ratio of the mass of the fluid to that of the structure. A traditional partitioned (TP) scheme using a Dirichlet–Neumann coupling for the same model problem is shown to be unconditionally unstable if the added mass of the fluid is too large. A series of benchmark problems of increasing complexity are considered to illustrate the behavior of the AMP algorithm, and to compare the behavior with that of the TP scheme. The results of all these benchmark problems verify the stability and accuracy of the AMP scheme. Results for one benchmark problem modeling blood flow in a deforming artery are also compared with corresponding results available in the literature.

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1. Introduction

Fluid–structure interaction (FSI) problems that describe the motion of an incompressible fluid coupled to a thin-walled structure (beam or shell) arise in many applications such as those in structural engineering and biomedicine. Such FSI problems are often modeled mathematically by suitable partial differential equations for the fluids and structures in their respective domains, together with matching conditions at the boundaries of the domains where the solutions of the equations interact. Numerical algorithms used to solve these FSI problems can be classified into two main categories. Algorithms belonging to one category, called monolithic schemes, treat the equations for the fluids and structures along with interface and boundary conditions as a large system of evolution equations, and then advance the solutions together. The other category of algorithms are partitioned schemes (also known as modular or sequential schemes), and these algorithms employ separate solvers for the fluids and structures which are coupled at the interface. Sub-iterations are often performed at each time step of partitioned algorithms for stability. Even though many existing partitioned schemes suffer from moderate to serious stability issues in certain problem regimes, they are often preferred since they can make use of existing solvers and can be more efficient than monolithic schemes.

The traditional partitioned algorithm for beams (or shells) uses the velocity and/or acceleration of the solid as a boundary condition on the fluid. The force of the fluid is accounted for through a body forcing on the beam. It has been found that partitioned schemes may be unstable, or require multiple sub-iterations per time step, when the density of the structure is similar to or lighter than that of the fluid [1,2]. These instabilities are attributed to the *added-mass effect* whereby the force required to accelerate a structure immersed in a fluid must also account for accelerating the surrounding fluid. The added-mass effect has been found to be especially problematic in many biological flows such as haemodynamics since the density of the fluid (blood) is similar to that of the adjacent structure (arterial walls) [3].

In our previous companion papers [4,5], we developed stable added-mass partitioned (AMP) algorithms for linearized FSI problems involving incompressible Stokes fluids coupled to elastic bulk solids and to beams. The key ingredient of our approach is the use of non-traditional Robin interface conditions. These condition are derived at a continuous level, have no adjustable parameters, and are amenable for incorporation in high-order accurate schemes. Using mode analysis, the AMP approach was shown to be stable without sub-iterations per time step, and the numerical results demonstrated second-order accuracy in the max-norm. In the present paper, we focus on the case of thin structures and our primary purpose is to extend the AMP scheme in [5] to the case of finite amplitude motions in more general geometries. In particular, the AMP interface conditions are derived for beams of finite thickness, for beams with fluid on two sides, and for beams with a free end immersed in the fluid. Finite amplitude motions of the beam results in finite deformations of the surrounding fluid domain. Our numerical approach for the solution of the fluid equations in an evolving fluid domain is based on the use of deforming composite grids (DCG). The DCG approach for FSI problems was described first in [6] for inviscid compressible flow coupled to a linearly elastic solid, and later in [7] for compressible flow coupled to nonlinear hyperelastic solids. The approach is extended here for incompressible flow coupled to beams. Both finite-difference and finite-element approximations to the equations governing the motion of the beam are considered, and issues concerning the interface coupling of a finite-element based beam solver to a finite-difference based fluid solver are discussed. The stability analysis in [5] is also extended to treat beams with fluid on two sides. Finally, several benchmark FSI problems are presented to illustrate the stability and accuracy of the present AMP algorithm.

The investigation of FSI problems and the development of numerical approximations for their solution are very active areas of research, see for example [8–10] and the references therein. For the coupling of incompressible flows and bulk solids, the work in [3,4,10–12] describes recent developments. The development of partitioned schemes for the case of incompressible fluids coupled to thin structures, the focus of this paper, is also an active area. The first stable partitioned scheme for incompressible flow coupled to thin structures was the "kinematically coupled scheme" of Guidoboni et al. [13] which uses an operator splitting of the kinematic interface condition (matching the fluid and beam velocities). This scheme was further advanced in [14,15]. Lukáčová-Medvid'ová et al. [16] developed a partitioned scheme based on the kinematically coupled scheme that uses Strang splitting. Fernandez and collaborators have also developed stable partitioned schemes for incompressible flow coupled to thin structures that are based on a time-splitting of the kinematic boundary condition [17–22]. However, it appears to be difficult to achieve higher than first-order accuracy with the time-splitting approach. The AMP approximation, in contrast, is not based on a time splitting and is amenable to second- or even higher-order accuracy. Second-order accuracy in the max-norm was demonstrated in [5] for linearized problems, while in this paper second-order accuracy is shown for the full nonlinear problem with deforming domains.

The remainder of the paper is organized as follows. In Section 2 we describe the governing equations. The AMP interface conditions are derived in Section 3. A second-order accurate predictor–corrector algorithm based on these conditions is described in Section 4. A specific choice for the beam model is given in Section 5. In Section 6, the stability of the AMP scheme is shown for a linearized FSI problem involving a beam with fluid on two sides. The numerical approach used for moving domains based on deforming composite grids, our numerical approaches for the beam solver, and the treatment of the AMP interface conditions are described in Section 7. Numerical results presented in Section 8 carefully demonstrate the stability and accuracy of the AMP algorithm. Conclusions are provided in Section 9.