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A new approach to robust, weighted signal averaging

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ABSTRACT

In this paper, a new approach for robust, weighted averaging of time-aligned signals is proposed. Suppression of noise in such case can be achieved with the use of the averaging technique. The signals are time-aligned and then the average template is determined. To this end, the arithmetic mean operator is often applied to the synchronized signal samples or its various modifications. However, the disadvantage of the mean operator is its sensitivity to outliers. The weighted averaging operation can be regarded as special case of clustering. For that reason in this work the averaging process is formulated as the problem of certain criterion function minimization and a few different cost functions are employed. The maximum likelihood estimator of location based on the generalized Cauchy distribution is used as the cost function. Such approach allows to suppress various types of impulsive noise. The proposed methods performance is experimentally evaluated and compared to the reference methods using electrocardiographic signal in the presence of the impulsive noise and the real muscle noise as well as the case of noise power variations.

1. Introduction

Averaging is one of the basic methods in statistical analysis of experimental science, especially in the case when the analyzed system response is time-aligned or (quasi)periodic [1]. A particular reason for application of averaging is the following: the traditional linear filtering schemes fail when the signal and noise frequency spectra significantly overlap [2]. This situation often takes place in analysis of biomedical signals, such as electrocardiograms (ECG), electroencephalograms (EEG) or other ones. Unfortunately, the arithmetic averaging is affected by a quite serious drawback which is its sensitivity to outliers, such as by spike artifacts or bursts of noise in the averaged signal. For these reasons the weighted robust averaging should be applied [3].

In order to perform a signal averaging of a such quasi-periodic signal like ECG signal the following preparations should be performed. The first step is the QRS detection procedure [4], then the signal is divided into periods containing a P-QRS-T wave and finally, the precise alignment according to the fiducial point (e.g. R-wave) of ECG period is performed [5]. If the localization of QRS complexes before signal averaging is not precise enough, the resulting signal will

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suffer from a low-pass filtering effect which may hide relevant high-frequency low-level potentials [5]. Another case is sensory evoked potentials (EPs) which are time-aligned. In clinical practice the auditory (AEPs), visual (VEPs) and somatosensory (SEPs) are tested most frequently [6,7]. The most common method to isolate the EPs from background EEG is averaging the EEG responses to multiple identical stimulations (visual or auditory). The EP can be regarded as the deterministic component for all stimuli (because it is time-locked) while the background noise (including spontaneous EEG) will vary and thus be reduced by averaging [8].

The idea of weighted averaging appears in various forms and the general assumption is that each observation influences the composite signal according to its weight, and aggregation can be viewed as an optimization problem over the vector of weights, using an objective function that measures dissimilarities between the resulting prototype and each observation [9–11]. At such point of view, the weighted averaging is similar to the fuzzy robust clustering methods [12,13]. Certain modifications of this idea, which are based on partitioning of input data set in the time domain and Bayesian approach, are presented in [14]. An important limitation of weighted averaging comes from the assumed noise model. According to this model, noise is varied between periods but is stationary inside each period [3].

Frequently the noise components have impulsive nature which is quite different from that of the Gaussian noise. In biomedical signal processing, it is the muscle noise that often has impulsive nature. Non-gaussianity results in significant performance degradation for systems optimized under the assumption of gaussianity [15]. The influence of outliers on the calculated average signal can be reduced by choosing a median as the aggregation operation. However the median averaging does not only remove the outliers but also the rest of data [3,9]. It means that some useful information can be lost. The alternative approach is based on the trimmed mean method in which a part of extreme values is set aside or modified and all other values are used for averaging, in the same way as in conventional mean averaging [3]. The possibility of using the myriad cost function to develop a procedure for robust weighted averaging is presented in [16]. The approach based on L_p-norm is presented in [17]. An application of Vapnik ε-insensitive function allows to increase the robustness of the weighted averaging is presented in [9].

The paper presents a family of robust cost functions, based on the Generalized Cauchy distribution, which are applied to develop a new approach to robust weighted averaging methods based on the criterion function minimizing. The proposed approach allows to create at least two special cases of the cost functions, i.e. the myriad and the meridian ones. In this work the proposed methods are applied to ECG signal averaging. There is also presented performance comparison of all described method using the ECG cycle obtained from the PTB diagnostic ECG database [18,19] as well as the muscle artifacts from the MIT-BIH noise stress test database (MA record) [19,20] and the synthetic impulsive noise. The paper is divided into four sections. Section 2 presents the idea of the weighted averaging based on the minimization of the scalar criterion function, and introduces the method proposed. In Section 3 numerical experiments are performed. The final conclusions are specified in Section 4.

2. Methods

2.1. *Criterion function minimization*

The idea of the weighted averaging based on criterion function minimization is the following [9]. Let us consider N time-aligned signals where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{iM}]^T$ is the ith time-aligned signal which consists of M samples and $1 \le i \le N$. Let $\mathbf{v} = [v_1, v_2, \ldots, v_M]^T$ is the averaged signal and $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ is the weight vector which satisfies the following condition:

$$\underset{1\le i\le N}{\forall} \quad w_i \in [0,1], \quad \sum_{i=1}^{N} w_i = 1. \tag{1}$$

The scalar criterion function is defined as [9,12]:

$$I_m(\mathbf{w},\mathbf{v}) = \sum_{i=1}^{N}\sum_{j=1}^{M}(w_i)^m \rho(z_{ij}), \tag{2}$$

where $z_{ij} = x_{ij} - v_j$, $m \in (1,\infty)$ is the assumed weighting exponent and $\rho(\cdot)$ is a measure of dissimilarity for the vector argument. Eq. (2) can be rewritten as:

$$I_m(\mathbf{w},\mathbf{v}) = \sum_{i=1}^{N}(w_i)^m \left(\sum_{j=1}^{M}\rho(z_{ij})\right), \tag{3}$$

where $\rho(\mathbf{z}_i) = \sum_{j=1}^{M}\rho(z_{ij})$ and $\mathbf{z}_i = \mathbf{x}_i - \mathbf{v}$. Expression (3) forms the objective function by summing the dissimilarity over all points. Then the scalar criterion function (3) can be formulated as [9]:

$$I_m(\mathbf{w},\mathbf{v}) = \sum_{i=1}^{N}(w_i)^m \rho(\mathbf{z}_i). \tag{4}$$

The function $I_m(\mathbf{w},\mathbf{v})$ is regarded as a measure of total dissimilarity between $\mathbf{v}$ and signal cycle $\mathbf{x}_i$, weighted by $(w_i)^m$. The task of searching for an optimal averaged signal $\mathbf{v}^*$ and an optimal weight vector $\mathbf{w}^*$ can be formulated as follows:

$$I_m(\mathbf{w}^*,\mathbf{v}^*) = \min_{\mathbf{w},\mathbf{v}} \; I_m(\mathbf{w},\mathbf{v}). \tag{5}$$

The problem of I_m minimization is considered as the constrained optimization problem. If $\mathbf{v}$ is fixed, then the Lagrangian functional of (4) with the constraint from (1) is [9]:

$$L(\mathbf{w},\lambda) = \sum_{i=1}^{N}(w_i)^m \rho(\mathbf{z}_i) - \lambda\left[\sum_{i=1}^{N} w_i - 1\right], \tag{6}$$

where λ is the Lagrange multiplier. In order to solve formula (6) with respect to λ and the weights set $\mathbf{w}$, the Lagrangian's gradient is set to 0. This way we obtain:

$$\frac{\partial L(\mathbf{w},\lambda)}{\partial \lambda} = \sum_{i=1}^{N} w_i - 1 = 0 \tag{7}$$

and

$$\underset{1\le j\le N}{\forall} \quad \frac{\partial L(\mathbf{w},\lambda)}{\partial w_j} = m(w_j)^{m-1}\rho(\mathbf{z}_j) - \lambda = 0. \tag{8}$$