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Optical tomography reconstruction algorithm with the finite element method: An optimal approach with regularization tools

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ABSTRACT

Optical tomography is mathematically treated as a non-linear inverse problem where the optical properties of the probed medium are recovered through the minimization of the errors between the experimental measurements and their predictions with a numerical model at the locations of the detectors. According to the ill-posed behavior of the inverse problem, some regularization tools must be performed and the Tikhonov penalization type is the most commonly used in optical tomography applications. This paper introduces an optimized approach for optical tomography reconstruction with the finite element method. An integral form of the cost function is used to take into account the surfaces of the detectors and make the reconstruction compatible with all finite element formulations, continuous and discontinuous. Through a gradient-based algorithm where the adjoint method is used to compute the gradient of the cost function, an alternative inner product is employed for preconditioning the reconstruction algorithm. Moreover, appropriate re-parameterization of the optical properties is performed. These regularization strategies are compared with the classical Tikhonov penalization one. It is shown that both the re-parameterization and the use of the Sobolev cost function gradient are efficient for solving such an ill-posed inverse problem.

1. Introduction

Among the new imaging modalities expected to be available in the future, optical tomography is one of the most promising. It is used in flow diagnostics, medical imaging, food processing, etc. This laser-based probing technique may be divided into direct imaging where the emerging signal is directly used for projection and the reconstruction imaging which is based on the solution of an inverse problem. For both of them, recent research tends to show that the use of the long term photons, which have travelled for a long time in the whole sample to be probed, generates more information to the image reconstruction [1].

In direct tomography, a measurable variable of the transmitted or/and reflected signals is processed in order to extract some information about the inside of a semi-transparent medium on which a laser beam has been applied. In material with a high level of scattering, direct tomography is of limited use because photons do not progress along a straight line and the reconstruction is therefore non-linear, which prohibits the use of direct reconstruction methods such as the Radon transform method [2]. The other method, also called optical tomography, is an inverse-based reconstruction technique where the

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optical properties are estimated from boundary measurements of transmitted light, providing a non-invasive diagnostic tool for medical applications as the optical properties are related to the pathological or physiological state of tissues. The reconstruction is done by minimizing a cost function that measures the errors between the experimental measurements at the detectors and their prediction with a numerical model [1,3].

Major improvements have been carried out in the last decades using the full radiative transfer equation (RTE) as this equation well describes light transport in biological tissues. Then, different forms of the RTE have been used [1,4–6] among which the frequency domain formulation is the most used. The frequency domain approach provides an alternative to scientists to avoid the technical limitations intrinsic to the use of the time-domain approach. It allows a better separation of the optical properties by giving some additional information (phase shift) compared to the stationary domain [7]. Also, highly accurate numerical formulations of the RTE have been achieved such as the discontinuous Galerkin finite element formulation [8–10]. This method uses numerical fluxes to achieve local conservativity [11–13] compared to continuous finite element formulations that suffer from the lack of local conservativity.

The ill-posed behavior of the inverse problem requires the use of accurate forward models of light transport coupled with robust optimization techniques. Also, the noise which is inherent by nature to the measurements leads to a non-smooth gradient of the cost function when the classical L_2 inner product is used within the adjoint method and thus the related optimization procedure may be slow and less accurate. In addition, parameters can be of different orders of magnitude, which often leads to problems of crost-talk. Generally, the method of Tikhonov regularization is used to reduce these difficulties. This technique is based on a penalization of the difference between the obtained properties and some guessed ones chosen a priori (usually the background) within the cost function to be minimized. This old regularization method [14] which provides information to the inverse problem actually stabilizes it. However, the choice of the weigh related to the penalization term is problematic since it often leans on the search of a particular region on the so-called L-curve [15]. The use of alternative inner products when extracting the gradient of the cost function, as initiated by [16], aims at smoothing the gradient and acts as a preconditionner for the reconstruction optimization problem. Also, a re-parameterization of the functional space related to the optical properties is performed in order to avoid over-parameterization with respect to the lack of measurement information.

Then, this paper focuses on new strategies to improve the reconstruction in optical tomography by using the Sobolev gradient (gradient filtering) with finite element parameterization (mesh and space approximation) of the optical parameters. For this purposes, an optimization of the reconstruction scheme is introduced through the choice of an inner product within the adjoint method for the computation of the cost function gradient. The adjoint equations is derived from the continuous radiative transfer equations (CRTE). This leads, when choosing different discretization schemes, rather than reconsidering the adjoint equations, to choose a numerical scheme for the adjoint problem that is coherent with the one chosen for the forward problem. We thus chose the so-called "Differentiate-then-Discretize" approach as opposed to the "Discretize-then-Differentiate" approach as defined in [17] for the simplicity and conciseness when deriving the adjoint equations, and especially because we make use of different functional spaces (different meshes) for the states, the adjoint states, the optical properties, etc.

The paper is organized as follows. Section 2 presents the forward model equations describing the radiative transfer equation along with the measurable quantity used for optical tomography purposes. Section 3 states the cost function that is to be minimized, writes down the optimization problem and describes carefully the adjoint problem based on the continuous radiative transfer equation along with the cost function directional derivatives. Section 4 gives some specific tools that are to be used to cope with the ill-posed nature of the inverse problem, i.e., the use of regularization. Specially, the classical Tikhonov regularization is presented with its pros and cons. Other strategies such as the use of an appropriate finite element (mesh and space) parameterization of the optical properties and the use of the Sobolev gradient instead of the usual Hilbert one are of interest. Numerical tests are performed on the presented regularization tools with a comparative analysis. Especially, the use of the Sobolev Gradient and the appropriate re-parameterization is compared with the classical use of the Tikhonov regularization. The last section deals with the conclusions and extended future work.

2. Forward model equations

2.1. Model equations

In optical tomography, the forward model is a numerical model of light transport within the tissues. It aims at computing the prediction of the measurements at the detectors once the source and the optical properties of the medium are known. This model is described by a Boltzmann type integro-differential equation called the radiative transfer equation [1,18]. This equation is difficult to solve and analytical solutions are available only for simple cases. Below we present the equations of the forward model and the measurement prediction.

The forward model used is this study is the frequency domain form of the radiative transfer equation which writes [3]:

$$\left(\vec{\Omega}\cdot\nabla+\frac{i\omega}{c}+\kappa+\sigma\right)I(r,\vec{\Omega},\omega)=\frac{\sigma}{4\pi}\int_{4\pi}I(r,\vec{\Omega}',\omega)\Phi(\vec{\Omega}',\vec{\Omega})\mathrm{d}\Omega' \tag{1}$$