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A fast multigrid-based electromagnetic eigensolver for curved metal boundaries on the Yee mesh

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ABSTRACT

For embedded boundary electromagnetics using the Dey–Mittra (Dey and Mittra, 1997) [1] algorithm, a special grad–div matrix constructed in this work allows use of multigrid methods for efficient inversion of Maxwell's curl–curl matrix. Efficient curl–curl inversions are demonstrated within a shift-and-invert Krylov-subspace eigensolver (open-sourced at [ofortt]https://github.com/bauerca/maxwell[cfortt]) on the spherical cavity and the 9-cell TESLA superconducting accelerator cavity. The accuracy of the Dey–Mittra algorithm is also examined: frequencies converge with second-order error, and surface fields are found to converge with nearly second-order error. In agreement with previous work (Nieter et al., 2009) [2], neglecting some boundary-cut cell faces (as is required in the time domain for numerical stability) reduces frequency convergence to first-order and surface-field convergence to zeroth-order (i.e. surface fields do not converge). Additionally and importantly, neglecting faces can reduce accuracy by an order of magnitude at low resolutions.

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1. Introduction

The Dey–Mittra electromagnetics algorithm simulates smooth curved perfectly-conducting boundaries using the Yee finite-difference technique [3,1]. The algorithm is often called a cut-cell or embedded-boundary technique since the mesh does not conform to the geometry of the conducting boundary (grid cells, faces, and edges are "cut" by boundaries). In the time-domain, the Courant–Friedrichs–Lewy (CFL) condition reduces the accuracy of the Dey–Mittra algorithm by requiring the neglecting of some cut faces. More precisely, the CFL condition states that the maximum stable timestep is limited by the maximum eigenvalue (of the discretized curl–curl matrix) and, in the Dey–Mittra algorithm, the maximum eigenvalue can be inflated greatly by faces barely cut by a boundary. A trade-off between accuracy and wall-clock simulation time ensues; if fewer neglected faces are desired (greater accuracy), the time-step must be reduced [1,2]. In this paper, we consider the Dey–Mittra algorithm in the frequency-domain, where the CFL condition does not apply and the full accuracy of the method can be used.

We begin by reviewing the two important aspects of the problem: (1) the Dey–Mittra algorithm and (2) eigensolving Maxwell's equations as discretized on the Yee mesh. (Ultimately, this leads to the question: How does one invert the curl–curl operator? Fortunately, this is well-studied [4–9].) The advance of this paper is described in Section 5, and amounts

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to a transformation of the discretized Dey–Mittra curl–curl operator that allows efficient inversion by multigrid techniques [10]. Proof of performance is given in the numerical results, where our eigensolver attacks the spherical resonant cavity and the 9-cell TESLA superconducting accelerator cavity. The code used throughout this paper is open-sourced, and can be found at https://github.com/bauerca/maxwell.

2. The Dey–Mittra algorithm

Electromagnetic cavity eigenmodes are solutions to Maxwell's wave equation subject to perfectly conducting boundary conditions; a magnetic eigenmode satisfies

$$\nabla \times \nabla \times \mathbf{B} = k^2 \mathbf{B} \quad \text{in } \Omega, \tag{1}$$

$$\mathbf{n} \cdot \mathbf{B} = 0 \quad \text{on } \partial\Omega, \tag{2}$$

where Ω is the cavity interior, $\partial\Omega$ is the perfectly conducting boundary, $\mathbf{n}$ is the normal to the boundary, and $k = \omega/c$, where ω is the resonant angular frequency and c is the speed of light. We discretize Maxwell's equations with the finite-difference Yee algorithm [3], labeling the grid electric and magnetic field components as $e_{\alpha|ijk}$ and $b_{\alpha|ijk}$, respectively, where α is one of x, y, or z and i, j, and k are integer grid cell indices. Fig. 1 shows the spatially staggered component layout of the Yee scheme which ensures the first-order accuracy (second-order error) of the discretized curl operators. In matrix–vector form, where $\mathbf{b}$ ($\mathbf{e}$) is the vector of all $b_{\alpha|ijk}$ ($e_{\alpha|ijk}$) components, the discretized version of Eq. (1) in vacuum is written [11,12]

$$\mathrm{C}\mathrm{C}^{\mathrm{T}}\mathbf{b} = k^2\mathbf{b}. \tag{3}$$

The Yee layout guarantees that the curl of the electric field is the transpose of the curl of the magnetic field, resulting in the symmetric matrix of Eq. (3) (the curl–curl matrix is also positive semi-definite, i.e. $k^2 \geqslant 0$).

The Dey–Mittra algorithm is a modification of the Yee algorithm which simulates curved perfectly conducting boundaries in 3D with second-order error [1,2]. The algorithm is based on the finite integral interpretation of the Yee algorithm [13,14] where, for example, the Yee Faraday update for $b_{x|ijk}$ (in the frequency domain) is written as

$$-i\omega b_{x|ijk} = \frac{1}{a_{x|ijk}}\left(l_{y|ijk}e_{y|ijk} - l_{y|ijk+1}e_{y|ijk+1} + l_{z|ij+1k}e_{z|ij+1k} - l_{z|ijk}e_{z|ijk}\right), \tag{4}$$

which is a representation of Faraday's Law in integral form: $-i\omega\int \mathbf{B}\cdot d\mathbf{a} = \oint \mathbf{E}\cdot d\mathbf{l}$. In the above, $l_{\alpha|ijk}$ is the length of the edge of the Yee grid cell on which the component, $e_{\alpha|ijk}$, is centered (see Fig. 1). Similarly, $a_{\alpha|ijk}$ is the area of the cell face on which the component, $b_{\alpha|ijk}$, is centered. In vacuum, $l_{x|ijk} = \Delta x$, $l_{y|ijk} = \Delta y$, and $a_{x|ijk} = \Delta y\Delta z$ such that Eq. (4) reduces to the usual Yee finite difference expression.

When a face, $a_{\alpha|ijk}$ is intersected by a perfectly conducting boundary, the Dey–Mittra algorithm takes $l_{\alpha|ijk}$ and $a_{\alpha|ijk}$ to be the portion of the length and area, respectively, *outside* the conductor (see Fig. 2). This is a physically meaningful representation of Faraday's Law in integral form since the electric field tangent to conducting boundaries vanishes. In matrix–vector form, the Dey–Mittra algorithm changes Eq. (3) to

$$\mathrm{A}^{-1}\mathrm{C}\mathrm{L}\mathrm{C}^{\mathrm{T}}\mathbf{b} = k^2\mathbf{b}, \tag{5}$$

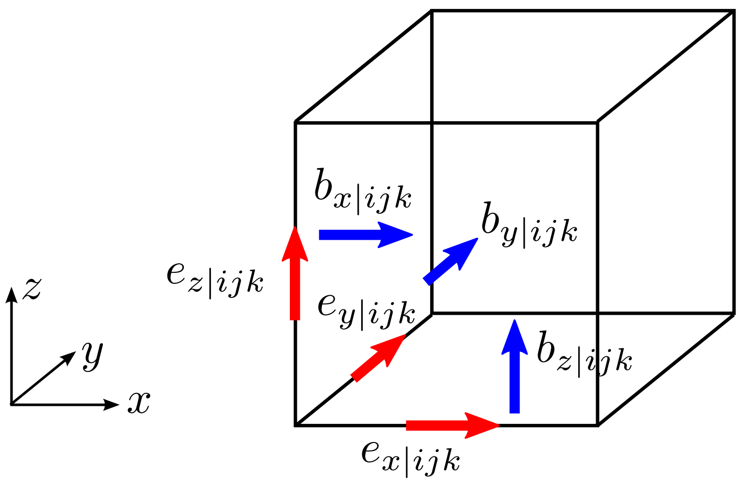

Fig. 1. Yee grid cell *ijk*. Electric (magnetic) field components are centered on edges (faces).

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