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A Fully Eulerian formulation for fluid–structure-interaction problems

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ABSTRACT

In this work, we present a Fully Eulerian framework for fluid–structure interaction (fsi) problems coupling the incompressible Navier–Stokes equations with a hyperelastic solid.

The Fully Eulerian framework is a monolithic implicit variational formulation for the coupled problem. In contrast to the well-established Arbitrary Lagrangian Eulerian (ALE) coordinates, the Fully Eulerian framework formulates both subproblems, fluid and solid, in Eulerian coordinates. This concept circumvents various difficulties connected to ALE coordinates since no artificial domain mapping is used. The formulation is an interface-capturing method and uses an extension of the solid's deformation, the *Initial Point Set*, to detect the interface location.

By construction, very large deformation as well as topology changes like contact of the solid to the domain boundary or other solid parts are possible.

1. Introduction

We present a monolithic variational finite element method for fluid–structure interaction problems. Emphasis is on applications where very large structural deformations, free movement of the structure within a flow domain and contact of the structure with the domain's boundary, with other structures or self-contact appears. The formulation presented in this work is Eulerian–Eulerian and a first variant of this novel approach has been suggested by Dunne [14,15].

There exist countless different approaches to model and simulate fluid–structure interaction problems. Among them, we focus on monolithic models, where the complete problem is formulated in one coupled system including the interface conditions between solid and fluid. Monolithic models allow for implicit solution schemes, large timesteps and offer the possibility to use sensitivity based error estimation and optimization methods. They are well suited for the simulation of problems with large fluid densities as appearing in hemodynamics [18]. While fluid problems are naturally described in a fixed Eulerian coordinate framework, a Lagrangian, material centered description is the usual basis for solid problems. All monolithic schemes for a fluid–solid interaction must somehow match these two different frameworks.

In Lagrangian or arbitrary Lagrangian methods, the flow problem is mapped onto a matching reference domain. Classical approaches are the ALE method, see [29,4,35] or deforming-spatial-domain/ stabilized space–time methods (DSD/SST), see for instance [53,51]. These formulations have in common, that kinematic and dynamic coupling conditions are easily embedded into trial spaces and established by variational techniques. A drawback of Lagrangian methods is the underlying transformation of the fluid-problem which can break down for large deformations or large solid movements. Lagrangian approaches are interface-tracking methods as the common interface is shared by both subproblems.

The Eulerian–Lagrangian methods use a Eulerian fixed computational mesh for the fluid problem and a Lagrangian mesh for the solid problems. Coupling of the two frameworks is accomplished by using force densities as in the immersed boundary method by Peskin [43] or the immersed interface method [39]. Other approaches introduce additional interface variables

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and couple the two systems using Lagrange multipliers. Examples are the fictitious domain method [22], or other recent approaches based on the extended finite element method [20]. A survey of interface coupling approaches is given by Felippa et. al. [17]. Eulerian–Lagrangian methods are interface-capturing approaches. The interface is not part of the fixed Eulerian fluid mesh and its location within the computational fluid domain must be captured implicitely as part of the solution scheme. An early approach for capturing the interface is the Volume of Fluid method [30], where a tracking function Φ_V takes the value one in the fluid domain and zero in the solid domain. This function Φ_V is transported with the interface velocity. A proper numerical approximation of Φ_V with a sharp jump from fluid to solid domain is one of the difficulties connected to this scheme. Another possibility for capturing the interface – at first used to model multiphase-flows – is by means of a scalar Level-Set [42,47] function Φ_L which in any point of the domain indicates the signed distance to the interface. The interface itself is given as zero-contour of the Level-Set function. Based on Level-Sets, very efficient and simple numerical schemes exist to describe free-boundary and fluid–structure interactions [38]. Usually, Level-Set representations of the interface lack the ability to reproduce sharp corners. A general problem of Eulerian–Lagrangian methods are the approximation properties close to the interface. Since the fluid elements are cut by the interface and the solution may be discontinuous (or at least not differentiable) across the interface, approximation with standard finite elements is difficult. Here, the extended finite element method helps to improve accuracy [11,20].

Finally, for problems with very large deformation, an Eulerian description of the structural problem is desirable [54,41]. Using a fixed Eulerian background mesh, the interface between fluid and solid will freely move in the domain. A Eulerian method will always be of interface-capturing type. Dunne [14] and Dunne et al. [15] has first implementations of an Eulerian-Eulerian model for the interaction of an incompressible fluid with an hyper-elastic solid. Here, capturing of the interface is accomplished with the Initial Point Set (IPS), a vector-field Φ_{IPS} used to transport the complete reference coordinate system. The IPS-method is able to capture interfaces with sharp edges. A finite difference approach for Eulerian–Eulerian fluid–structure interaction based on the Volume of Fluid method [30] is introduced by Sugiyama et al. [50]. Here, the coupling between incompressible fluid and structure is solved with a pressure correction iteration. An Eulerian-Eulerian fluid–structure interaction method based on Level-Set functions is introduced by He and Qiao [23]. Four Level-Set functions are required to represent the moving structure domain. Two of them take a role similar to the Initial Point Set as introduced by Dunne [14] and Dunne et al. [15].

The Fully Eulerian formulation introduced in this work is based on extensions of the IPS method [46,16,44]. Fluid and solid problem are given in an Eulerian formulation, the interface conditions are embedded in the function spaces and realized by variational load balancing. For capturing the interface the solid's deformation will be extended to only a small layer in the fluid domain. The flow problem is modeled without any transformation and apart from the small interface layer no additional variables must be introduced, making the method very efficient.

In the second section we shortly introduce the required notation. Section 3 is devoted to the governing equations in Eulerian coordinates and deals with the coupling of fluid and solid problem. Details on discretization and the solution scheme are provided in Section 4. Finally, in Section 5 we present different numerical examples demonstrating the scope of this new Fully Eulerian fluid–structure interaction formulation. We conclude in Section 6.

2. Preliminaries

Let $\Omega \subset \mathbb{R}^2$ be a two dimensional domain. At time $t = 0$ this domain is split into a non-overlapping partitioning into the fluid part $\hat{\Omega}_f := \Omega_f(0)$ and solid part $\hat{\Omega}_s := \Omega_s(0)$ with a common interface $\hat{\Gamma}_i := \partial\hat{\Omega}_f \cap \partial\hat{\Omega}_s$. We will consider problems, where the domain partitioning will change in time, the combined domain Ω however will be fixed:

$$\hat{\Omega}_f \mapsto \Omega_f(t), \quad \hat{\Omega}_s \mapsto \Omega_s(t), \quad \Gamma_i(t) = \partial\Omega_f(t) \cap \partial\Omega_s(t), \quad \Omega = \Omega_f(t) \cup \Omega_s(t).$$

At time $t = 0$ we call $\hat{\Omega}_f$ and $\hat{\Omega}_s$ the reference configuration. In the context of fluid–structure interaction, $\hat{\Omega}_s$ refers to the Lagrangian view-point and $\Omega_s(t)$ to the Eulerian. For simplicity, we will consider problems with Dirichlet boundary conditions only and define

$$\Gamma_f(t) := \partial\Omega_f(t) \cap \partial\Omega, \quad \Gamma_s(t) := \partial\Omega_s(t) \cap \partial\Omega.$$

As function spaces we use the Lebesgue space $L^2(\Omega)$ in the domain Ω or in the (moving) subdomains as well as Sobolev-spaces $H^m(\Omega)$ of L^2 functions with weak derivatives in L^2. By $(\cdot,\cdot)_\Omega$ we denote the L^2-inner product and by $\|\cdot\|_\Omega$ the usual L^2-norm on Ω. Further, by $\langle\cdot,\cdot\rangle_\Gamma$ we denote the L^2-inner product on (parts of) the boundary $\Gamma \subset \partial\Omega$, usually the inner interface between the two subdomains. By $H^1(\Omega)$ we denote the Lebesgue space of L^2-functions with first weak derivative in L^2 and by $H^1_0(\Omega;\Gamma)$ the space of H^1-functions with trace zero on (parts of) the boundary $\Gamma \subset \partial\Omega$.

3. Eulerian formulation of fluid structure interaction

In this section we derive the coupled system of equations describing the interaction of an incompressible Stokes or Navier–Stokes fluid with an elastic structure of St. Venant Kirchhoff type. All equations are given in variational formulation. This allows for an easy transformation between Eulerian and Lagrangian coordinate frameworks.