Journal of Computational Physics 231 (2012) 5351–5373

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Mitigating kinematic locking in the material point method

C.M. Mast [a], P. Mackenzie-Helnwein [a,*], P. Arduino [a], G.R. Miller [a], W. Shin [b]

[a] Department of Civil & Environmental Engineering, University of Washington, Box 352700, Seattle, WA 98195-2700, USA
[b] Shannon & Wilson, Inc., 400 North 34th Street, Suite 100, Seattle, WA 98103, USA

ARTICLE INFO

Article history:
Received 20 October 2011
Received in revised form 11 April 2012
Accepted 25 April 2012
Available online 14 May 2012

Keywords:
Material point method
Locking
Meshfree methods
Particle methods

ABSTRACT

The material point method exhibits kinematic locking when traditional linear shape functions are used with a rectangular grid. The locking affects both the strain and the stress fields, which can lead to inaccurate results and nonphysical behavior. This paper presents a new anti-locking approach that mitigates the accumulation of fictitious strains and stresses, significantly improving the kinematic response and the quality of all field variables. The technique relies on the Hu–Washizu multi-field variational principle, with separate approximations for the volumetric and the deviatoric portions of the strain and stress fields. The proposed approach is validated using a series of benchmark examples from both solid and fluid mechanics, demonstrating the broad range of modeling possibilities within the MPM framework when combined with appropriate anti-locking techniques and algorithms.

1. Introduction

The material point method (MPM) is a numerical technique that is used primarily for solving problems in which large, flow-like deformations occur. The method's effectiveness is due in part to the hybrid Eulerian–Lagrangian description of motion that lends itself nicely to applications in a wide variety of complex engineering problems, including those in both the solid and fluid mechanics regimes.

The formulation of the MPM is similar in many regards to the well known Finite Element Method (FEM). This association has, at times, proved beneficial—the MPM has been able to leverage existing knowledge and results that come along with a more established and well-studied technique. Conversely, several known shortcomings associated with the FEM have been inherited by the MPM. Perhaps the most notable of these shortcomings is the kinematic locking that can occur due to the interpolation functions constructed on the MPM background grid/mesh. In this context, the term *locking* refers to the build-up of fictitious stiffness that is due to an element/cell's inability to reproduce correct deformation mode shapes. The end result is a system that is too stiff, leading to poor kinematics and erroneous strains (and thus incorrect stresses).

In this paper, kinematic locking is identified within the confines of the MPM. It is shown that both volumetric and shear locking arise in the standard MPM algorithm when traditional, linear shape functions are used in conjunction with a regular, rectangular grid. An anti-locking algorithm based on the well known Hu–Washizu variational principle is proposed, and its effectiveness is highlighted using a series of benchmark examples from both fluid and solid mechanics. It is shown that with appropriate anti-locking techniques, the MPM can serve as a unified framework for analyzing systems exhibiting a full range of solid and fluid behavior.

* Corresponding author.
E-mail addresses: cmast@uw.edu (C.M. Mast), pmackenz@uw.edu (P. Mackenzie-Helnwein), parduino@uw.edu (P. Arduino), gmiller@uw.edu (G.R. Miller), geoshin@gmail.com (W. Shin).

0021-9991/$ - see front matter
http://dx.doi.org/10.1016/j.jcp.2012.04.032

The paper is organized as follows: Section 2 provides a brief overview of the MPM and its variants. A summary of our notation, as well as specific details of the MPM algorithm that are not new but helpful for the less familiar reader are given in Appendix A. The mathematical framework and the algorithmic development of the studied anti-locking strategies are given in Section 3, where both the theoretical background and the implementation aspects are given separate spaces. Section 4 provides a series of examples, designed to illustrate applicability of the proposed techniques to both fluid and solid applications, and to verify their accuracy through comparison to experimental data and analytical solutions. Section 5 contains a brief summary and conclusions.

2. Overview of the material point method

The MPM follows from the more general class of numerical schemes known as PIC, or Particle-In-Cell methods. The first PIC technique was developed in the early 1950s [21] and was used primarily for applications in fluid mechanics. Early implementations suffered from excessive energy dissipation, rendering them obsolete when compared to newer, more effective methods. In 1986, Brackbill and Ruppel solved many of the inherent problems related to energy dissipation and introduced FLIP—the Fluid Implicit Particle method [13]. The FLIP technique was modified and tailored for applications in solid mechanics by Sulsky et al. [51,54] and has since been referred to as the material point method [52].

The method was born out of a need to combine fluid-like, large deformations with a history-dependent material response. Significant research has focused on applications of this type (e.g., [11,12,17,34,59,61,40]), but much work has also been done in additional areas—pushing the capabilities of the MPM and exploring new applications. These include applications in geotechnical engineering [3,60,66], fracture and material failure [15,19,20,41,44,45,53,55], contact, material interaction, and penetration [6,7,23,24,27,33,38,46,42,64,65], as well as general implementation considerations [5,25,28,29,36,48,50].

The mathematical foundations and underpinnings of the technique have been explored in detail and are well documented in several publications, e.g., [54,6,29,14,18,35]. A detailed presentation of the numerical implementation for the standard algorithm can be found in these references. For notational convenience, key equations and algorithmic steps are summarized in Appendix A. The curious reader is encouraged to explore the supplied references for additional details regarding the standard algorithm.

In most implementations of the MPM, it is common to use standard linear shape functions defined on a regular, rectangular grid. This is not, however, a limitation or requirement of the technique. Researchers have demonstrated the benefits of using an irregular grid consisting of either triangular or quadrilateral cells, e.g., [59,57]. Others have investigated the use of higher-order shape functions—2nd and 3rd order B-splines [2,49], standard quadratic shape functions [2], as well as a Radial Basis Function [22]—in lieu of linear shape functions. In these cases the use of a higher order shape function eliminates many of the non-physical results associated with traditional implementations. However, the use of the non-linear functions can result in an increase in computational cost, potentially limiting the effectiveness of such approaches. Given the simplicity of low-order elements/cells, they remain a polular choice in FEM and MPM applications. To date, linear interpolation functions are the most common choice found in the MPM literature.

The use of linear shape functions does not come without shortcomings. In particular, the discontinuous gradient leads to potential *cell-crossing* errors, and *volumetric locking* due to the insufficient representation of an isochoric displacement/velocity field [10] and *shear locking* due to non-physical coupling of normal strains and shear strains [1]. The *cell-crossing* error is caused by the sudden jump in strain rate as a particle moves from one cell to another. A potential solution to the cell-crossing error is to counteract the discontinuity of the gradient at the interface between adjacent cells by introducing an enhanced gradient field [63]. Alternatively, the particle can be represented using a finite domain, effectively smoothing out the discontinuity. This idea was first developed in the Generalized Interpolation Material Point (GIMP) method [8] and has since been applied to wide variety of engineering problems, e.g., [16,30–32,56]. A similar variant of the MPM, the Convected Particle Domain Interpolation (CPDI) technique [43], effectively combats the discontinuous gradient issue by introducing a parallelogram-shaped particle domain that is consistently updated using the deformation gradient at the particle. Unfortunately these approaches do not address or alleviate kinematic locking.

In fact, until very recently [2,4,37,47], the topic has not been reported in the literature. The closest references are the works [2,22,49], which have found that representation and integration errors are reduced significantly by introducing higher order shape functions on the background grid. This is in agreement with related observations in the FEM, where increasing the interpolation order not only improves accuracy but also reduces locking phenomena.

Locking in the standard algorithm can be quite significant. Consider Fig. 1, in which two sample problems from fluid and solid mechanics (the two opposing ends of the materials spectrum) are used to highlight kinematic locking (see Section 5 for details regarding the MPM parameters, material parameters, etc., used for these analyses). Fig. 1(a) depicts a simplified dam break analysis in which a rectangular water column, initially in a hydrostatic state, flows to the right due to gravity. A short time later in Fig. 1(b), the water column has shifted but in no way represents a flowing mass of water. In addition to the poor kinematics, the stress state is relatively chaotic and differs by several orders of magnitude from the starting state. This large deformation analysis is a prime example of volumetric locking within the MPM. On the solid end of the materials spectrum, shear locking is prevalent as demonstrated by the shear stress distribution for the cantilever beam shown in Fig. 1(c) (it should be noted that the stress distribution alone is not necessarily an indication of locking, however, in this particular case, it is). This beam model is fixed on the left side, with prescribed initial velocity consistent with the first mode of vibration. The