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Short Note

Complement to the "Kolgan project"

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Earlier last year, one paper that originally appeared in Russian in the *Scientific Notes of TsAGI* was reprinted in English in the *Journal of Computational Physics* [1]. The reprint of the almost 40-years-old paper was made possible by Bram van Leer, a research scientist of worldwide reputation, who was bound in honour to correct the "historical oversight" relative to Vladimir Pavlovich Kolgan – the Soviet researcher who pioneered in the development of the high-resolution Godunov-type methods, now widely used in CFD, while his name and work stayed "completely unknown outside the USSR". In his accompanying notes [2], Bram van Leer gave a lot of interesting and useful concomitant information. Specifically, the author told about the live and work of Kolgan (data obtained from private communication), as well as about his own "search of Kolgan". He also highlighted the key ideas presented in [1], giving their detailed analysis in the historical context.

While rendering homage to Bram van Leer's great services and authority in computational science, and expressing my very grateful acknowledgment to him for realization of the "Kolgan project", I would like to seize the occasion to give some additional information and share the vision of Kolgan by the people coming from the same country.

1. First, a few words about Kolgan's renown in his native country. Nowadays his name could quite often be heard at CFD workshops and conferences in Russia, and his works are cited regularly in the domestic journals. In support of the aforesaid, one can refer to *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, a reputable Russian journal which is reissued in English under the name of *Computational Mathematics and Mathematical Physics*. In the last few years, Kolgan's works have been cited in this journal about twice a year – not so much, but not too bad for a broad-subject-area journal and the 40-years-old works. Given the numerous references in other periodicals and single publications, today the name of Kolgan just cannot go unnoticed by any Russian expert in CFD.

2. In 1975, two Kolgan's papers [3,4] dedicated to further development and application of the previously described scheme (Kolgan scheme) were published in the *Scientific Notes of TsAGI*.

In the first one, the scheme is generalized to solve 2D Euler equations; computations of supersonic flow over a flat step (infinite right-angle ledge) are described. Here, Vladimir Kolgan adds a new element to his scheme: the *minmod* limiter now

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involves the central difference as an adjunct to one-sided ones (left and right differences). This innovation can be commented as follows.

The central difference in the new algorithm is only selected in isolated points near local extrema (where its module is less than that of each one-sided difference). Theoretically, this has no effect on the conclusions made in [1] concerning the scheme stability and its monotonicity-preserving property. Presumably, Kolgan was dissatisfied with the function discontinuity in his original limiter. The new algorithm, while retaining the discontinuity, reduces substantially the jump.

Why did Kolgan not incorporate in his *minmod* limiter the so-called zero-branch that returns a zero value for the gradient if the two arguments have opposite signs? A probable reason may be Kolgan's reluctance to return to function constancy within a cell, even as an exception. In his works, he says nothing about the fact that the scheme accuracy in space drops to first order in some cells near local extrema. Probably, he simply overlooked this point and therefore tried to avoid the apparent fall-off in accuracy when employing the zero-branch. On that matter he had nothing to be grounded on – the works, where the TVD concept for high-order upwind methods was introduced and developed, appeared well later.

In this connection, one characteristic feature of the Kolgan scheme [1] can be indicated: the scheme is monotonicity-preserving if the CFL number $\nu \leqslant 2/3$; however, only the condition $\nu \leqslant 1/2$ guarantees its stability in the general case. Recall that in [1] Kolgan has proven rigorously the following property of the scheme as applied to the linear advection equation (hereafter the *forward* advection has to be borne in mind): after a single time step, each new cell average u_j^{n+1} will possess the value in between u_{j-1}^n and u_j^n, if $\nu \leqslant 1/2$ (note that if only the zero branch has been introduced in the limiter, the analysis of the same kind would relax this condition up to $\nu \leqslant 2/3$). It follows that the scheme is stable and monotonicity-preserving under such CFL condition. Kolgan stated as well that "a more involved analysis may be invoked to show that the scheme remains monotone for $\nu \leqslant 2/3$". This statement is clear for us, as the functions considered in proving the monotonicity-preserving property are monotonous and the zero-branch condition is not realized for them; therefore, in this case the Kolgan's minmod limiter is no different from the standard one. Consequently, the stability condition for monotonic functions is again $\nu \leqslant 2/3$. However, for nonmonotonic functions this extended condition is not valid. To evidence this claim, consider the following test case.

Let us start from the oscillatory data of the type: $u_j^n = a(-q)^j$, where a and q are the constants, $q > 1$. Then, for any subcell difference (a slope) the Kolgan's limiter yields $\Delta u_j = u_j - u_{j-1}$. Hence, for this test case the Kolgan scheme can be converted (using the relation $u_{j-1}^n = -u_j^n/q$) as follows:

$$u^j = u_j - \nu\left[\left(u_j + \frac{1}{2}\Delta u_j\right) - \left(u_{j-1} + \frac{1}{2}\Delta u_{j-1}\right)\right] = \left(1 - \frac{3}{2}\nu\right)u_j + 2\nu u_{j-1} - \frac{1}{2}\nu u_{j-2} = \left[1 - \nu\left(\frac{3}{2} + \frac{2}{q} + \frac{1}{2q^2}\right)\right]u_j \equiv bu_j$$

With this notation the stability requirement in our test case may be written as $|b| \leqslant 1$. Now, as q approaches unity, $b \to 1 - 4\nu$, and the stability requirement leads us to the condition $\nu \leqslant 1/2$, which proves the above claim.

Finally, remember that, once applied in a scheme of second-order of accuracy in both space and time (a MUSCL-type scheme), the Kolgan's limiter will not deteriorate the stability condition as compared to the standard minmod limiter or any familiar one. Furthermore, its perfect analogue has found use in the family of ENO schemes.

Now about the second of the above-mentioned papers. In [4], Kolgan applied his scheme for calculating a 3D problem of gas dynamics, namely, a steady flow over a body (a spherically blunt cylinder) under angle of attack. The calculations were done by the use of time-asymptotic technique on a fixed grid ($30 \times 17 \times 20$) becoming finer as the body surface is approached. To accelerate the convergence to a steady state, the author applied the technique that is now widely used. He described it as follows: "the calculation is made with the time transformation depending on the space size of the cell: each cell is computed using its own time step which is close to the maximum value for this cell; it makes possible to decrease significantly (nearly by an order of magnitude) the time of computing the problem". In addition, Kolgan reduced the number of dependent variables from five to four using the Bernoulli integral. As a result, the convergence to a steady state was achieved after 800 time steps; for BESM-6, the Soviet mainframe computer of the day, it took 12 h.

3. It took quite some time for Kolgan's works to gain wide recognition in the Soviet CFD community. Two reasons for this were indicated by van Leer in his accompanying notes [2]: (1) "the relative isolation of Kolgan's place of employment", and (2) the specific features of his personality – "there is no evidence that he (Kolgan) ever bothered to contact the mathematical establishment". These findings can be augmented with the reason of more general nature. It is by no means always the case that an advanced concept giving rise to a new trend in some field of science gains fast appreciation, especially if the author has not yet become an irrefutable authority, while there exist alternative lines of development. And in case of the Kolgan scheme such alternatives did exist. In support of this statement we can refer to one paper [5] by Minailos, a researcher from TsAGI, who was among the first to appreciate the merits of the Kolgan scheme and to begin to apply it. In 1977 he advocated the advantage of high-resolution monotone schemes based on the non-linear finite-difference approximation and ranked Kolgan's work [1] with the other works: [6–8] . Considering that Jay Boris first presented his FCT method in 1971 (for the relevant papers on FCT see van Leer's notes [2]), all the three works mentioned by Minailos predated Kolgan's work.

Further intensive development in the direction indicated by Kolgan resulted from the advantages of the Riemann-solver-based schemes that are well-known now. In this connection, it is pertinent to mention a book by Godunov (Ed.), Zabrodin, Ivanov, Kraiko and Prokopov [9] that was published in 1976. In this book, virtually all aspects of construction and application of the first-order Godunov scheme were considered in details. The success of this book in the Soviet CFD community contributed to popularization of the Godunov scheme, thus paving the way for further