Journal of Computational Physics 231 (2012) 4469–4498

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

A second-order accurate immersed boundary method for fully resolved simulations of particle-laden flows

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ARTICLE INFO

Article history:
Received 13 July 2011
Received in revised form 16 February 2012
Accepted 19 February 2012
Available online 30 March 2012

Keywords:
Immersed boundary method
Particle-laden flow
Finite-volume method
Grid locking
Multidirect forcing scheme
Retraction distance
Order of grid convergence

ABSTRACT

An immersed boundary method (IBM) with second-order spatial accuracy is presented for fully resolved simulations of incompressible viscous flows laden with rigid particles. The method is based on the computationally efficient direct-forcing method of Uhlmann [M. Uhlmann, An immersed boundary method with direct forcing for simulation of particulate flows, J. Comput. Phys. 209 (2005) 448–476] that is embedded in a finite-volume/pressure-correction method. The IBM consists of two grids: a fixed uniform Eulerian grid for the fluid phase and a uniform Lagrangian grid attached to and moving with the particles. A regularized delta function is used to communicate between the two grids and proved to be effective in suppressing grid locking. Without significant loss of efficiency, the original method is improved by: (1) a better approximation of the no-slip/no-penetration (ns/np) condition on the surface of the particles by a multidirect forcing scheme, (2) a correction for the excess in the effective particle diameter by a slight retraction of the Lagrangian grid from the surface towards the interior of the particles with a fraction of the Eulerian grid spacing, and (3) an enhancement of the numerical stability for particle–fluid mass density ratios near unity by a direct account of the inertia of the fluid contained within the particles. The new IBM contains two new parameters: the number of iterations N_s of the multidirect forcing scheme and the retraction distance r_d. The effect of N_s and r_d on the accuracy is investigated for five different flows. The results show that r_d has a strong influence on the effective particle diameter and little influence on the error in the ns/np condition, while exactly the opposite holds for N_s. A novel finding of this study is the demonstration that r_d has a strong influence on the order of grid convergence. It is found that for spheres the choice of $r_d = 0.3\Delta x$ yields second-order accuracy compared to first-order accuracy of the original method that corresponds to $r_d = 0$. Finally, $N_s = 2$ appears optimal for reducing the error in the ns/np condition and maintaining the computational efficiency of the method.

1. Introduction

The immersed boundary method (IBM) [21] has become a popular method for fully resolved simulations of particle-laden flows and flows through complex geometries more in general [19]. Characteristic for this method is that the computational grid for the fluid phase does not conform to the shape of the particles like in conventional methods with a body-fitted grid. Furthermore, the grid is typically structured, often Cartesian, fully continuous in space and fixed in time. The no-slip/

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http://dx.doi.org/10.1016/j.jcp.2012.02.026

no-penetration (ns/np) condition on the surface of a particle is not imposed explicitly, but instead additional forcing is applied to the flow in the immediate vicinity of the surface such that this condition is satisfied by good approximation. The advantage of the IBM over a method with a body-fitted grid is its computational efficiency: it does not require regridding when particles are moving and a simple structured grid enables the use of efficient computational methods for solving the Navier–Stokes equations. The price that has to be paid is a loss of accuracy because of the error in the approximation of the ns/np condition. The challenge is to develop an IBM that is both computationally efficient and sufficiently accurate.

Uhlmann [32] developed a computationally efficient IBM for particle-laden flows that is embedded in a finite-volume/pressure-correction method [35]. It makes use of two different grids: a fixed, uniform and continuous Cartesian grid for the fluid phase and a uniform grid attached to and moving with the surface of the particles. The present author will refer to the two grids as the Eulerian and the Lagrangian grid, respectively. The method solves the Navier–Stokes equations for the fluid phase and the Newton–Euler equations for the particles. The flow-induced force and torque acting on a particle is obtained from the IBM force distribution on the Lagrangian grid. The IBM force distribution on the Lagrangian grid is computed from the requirement that on the surface of the particle the prediction velocity of the pressure-correction scheme is equal to the local particle velocity. Since the grid points of the two grids do not overlap in general, interpolation is required of the prediction velocity from the Eulerian to the Lagrangian grid. Furthermore, spreading is required of the computed IBM force from the Lagrangian back to the Eulerian grid. In Uhlmann's IBM the interpolation and spreading operations [22] are based on the regularized Dirac delta function of Roma et al. [24] with a width of three Eulerian grid cells. Consequently, the particles have a smooth (i.e. non-sharp) interface from the point of view of the fluid phase.

The smoothing of the particle interface has the important advantage of suppressing undesired high-frequency oscillations in the force and torque acting on a particle when it moves over the Eulerian grid. These oscillations originate from variations in the interpolated prediction velocity when the Lagrangian grid moves with the particle over the Eulerian grid and thus changes its orientation relative to the Eulerian grid. In other words, they are present because the interpolation operation is not translation invariant [22]. This phenomenon is dubbed as *grid locking* by the present author, since the wavelength of the oscillations is set by the dimensions of the Eulerian grid cells and their period by the time it takes for a particle to travel from one Eulerian grid cell to another. Uhlmann [32] showed for the case of a forced oscillation of a cylinder in uniform cross-flow that the amplitude of the spurious oscillations decreases when the width of the regularized Dirac delta function is increased. The regularized Dirac delta function of Roma et al. [24] with a width of three Eulerian grid cells is considered as effective for suppressing grid locking and its compact support as computationally efficient [32].

The smoothing of the particle interface has, however, also an important disadvantage. The interpolation of the prediction velocity based on the regularized Dirac delta function of Roma et al. [24] is formally second-order accurate in space, but only when applied to a *smooth* velocity field. Peskin [22] pointed out that the velocity field near a solid boundary is *non-smooth* as it contains a jump in its normal derivative over the boundary. Consequently, the interpolation of the prediction velocity becomes first-order accurate and this explains the first-order spatial accuracy of Uhlmann's IBM for the flows studied in this paper. A possible way to improve the accuracy is to resort to a sharp representation of the interface. However, this will probably amplify grid locking [32], which is undesired. The challenge is to keep a smooth representation of the interface in order to suppress grid locking and to find other ways for improving the accuracy of the IBM.

In this paper a new IBM is presented that is based on the IBM of Uhlmann [32]. The objective of this study was to increase the numerical accuracy of the method and to improve its numerical stability for particle–fluid mass density ratios near unity, while maintaining its computational efficiency and the suppression of grid locking. Firstly, the approximation of the ns/np condition is improved by the multidirect forcing scheme of Luo et al. [17]. Secondly, a correction is implemented for the excess in the effective particle diameter by a slight retraction of the Lagrangian grid from the surface towards the interior of a particle [11]. The excess in the effective particle diameter is a direct consequence of the smoothing of the particle interface, which causes that the effective particle diameter is larger than the actual particle diameter from the point of view of the fluid phase. Thirdly, the numerical stability of the method is enhanced for particle–fluid mass density ratios near unity by a direct account of the inertia of the fluid contained within the particles [15].

The new IBM contains two new control parameters: the number of iterations N_s of the multidirect forcing scheme and the retraction distance r_d. The effect of N_s and r_d on the numerical accuracy has been investigated in detail for five different particle-laden flows. The main novelty of the present paper is the demonstration that for an appropriate choice of r_d the original IBM becomes second-order accurate in space, while the smooth representation of the particle interface still suppresses grid locking and the computational efficiency of the method is nearly unchanged.

This paper is organized as follows. The governing equations for particle-laden flows are given in Section 2. This is followed by a description of the IBM of Uhlmann [32] in Section 3. Next, the three main improvements to this method are presented in Section 4. The details of the numerical method are given in Section 5. The computational results are presented in Section 6. Finally, the conclusions and a discussion are given in Section 7.

2. Governing equations

Particle-laden flows are described by the Navier–Stokes equations for the fluid phase and the Newton–Euler equations for the solid particles.