Journal of Computational Physics 230 (2011) 8010–8027

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Ambipolar diffusion and drift in computational weakly-ionized plasmadynamics

Bernard Parent [a,*], Sergey O. Macheret [b,1], Mikhail N. Shneider [b]

[a] Dept. of Aerospace Engineering, Pusan National University, Busan 609-735, South Korea
[b] Dept. of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544-5263, USA

ARTICLE INFO

Article history:
Received 18 February 2011
Received in revised form 5 July 2011
Accepted 5 July 2011
Available online 20 July 2011

Keywords:
Ambipolar diffusion
Ambipolar drift
Convergence acceleration
Computational Fluid Dynamics (CFD)
Magnetohydrodynamics (MHD)
Weakly-ionized plasma
Quasi-neutral plasma
Multicomponent plasma

ABSTRACT

Modeling of ambipolar diffusion and drift taking place within a weakly-ionized fluid can lead to some convergence difficulties when the ion conservation equation and the electric field potential equation are solved consecutively. A novel formulation of the ion flow rate is proposed here that reduces the computing effort to reach convergence by a factor of 10 or more. It is shown that by recasting the ion flow rate in terms of drift and ambipolar diffusion components, the sensitivity to the electric field is reduced hence alleviating the stiffness of the system of equations and permitting significantly faster convergence. What makes the method particularly appealing is that (i) it yields faster convergence without affecting the accuracy of the converged solution and (ii) it is not restricted to specific discretization or relaxation schemes and can hence be readily implemented in existing flow solvers. Because it is developed in general form (*i.e.* applicable to a multicomponent plasma in the simultaneous presence of electric current and magnetic and electric fields), the method is notably well-suited to simulate ambipolar diffusion within ionized multi-species flow solvers and is recommended for all flowfields as long as the plasma remains weakly-ionized and quasi-neutral.

1. Introduction

Ambipolar diffusion is a phenomenon that occurs within a plasma when a spatial gradient of the number density of at least one of the charged species is present. The phenomenon is caused by the different charged species having different diffusivities, hence resulting in some of the charged species diffusing more or less rapidly than the others, which would lead to a loss of neutrality of the plasma. A minor loss of neutrality, however, induces an ambipolar electric field which, if the Debye length is sufficiently small, slows down the fast-diffusing species and speeds up the slow-diffusing species in such a way that the plasma remains quasi-neutral.

For a non-magnetized plasma composed of electrons and only one type of positive ions, a straightforward derivation shows that the effect of the ambipolar electric field causes the diffusion coefficient of the ions to be augmented according to the following expression (see for instance Ref. [1, p. 187]):

$$D_a = \left(1 + \frac{T_e}{T_i}\right) D_i \tag{1}$$

* Corresponding author.
E-mail addresses: parent@pnu.edu, bernparent@gmail.com (B. Parent).
URL: http://www.bernardparent.com.
[1] Current address: Lockheed Martin Aeronautics Company, 1011 Lockheed Way, Palmdale, CA 93599-0160, USA.

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2011.07.006

where D_i is the molecular diffusivity of the ions in the neutrals, D_a is the ambipolar diffusion coefficient, T_e is the electron temperature, and T_i is the temperature of the ions. From the latter, it is apparent that ambipolar diffusion effects would be particularly important when the plasma is in thermal non-equilibrium (that is, when the electron temperature differs significantly from the bulk gas temperature). Significant thermal non-equilibrium is a common feature of weakly-ionized plasmas under strong applied electromagnetic fields. For instance, in plasma aerodynamics [2–4] and in airborne MHD power generation [5] or MHD acceleration [6], it is not uncommon for the electron temperature to reach values in excess of 20,000 K while the bulk gas temperature remains at 300–600 K. For such applications, the ambipolar diffusion coefficient would be almost 100 times greater than the molecular diffusion coefficient, and would approach or even exceed the turbulence eddy diffusion coefficient. It follows that, in order to properly predict weakly-ionized flowfields under strong applied fields, ambipolar diffusion effects should be modeled as accurately as possible.

The simple expression presented in Eq. (1) is not appropriate for ambipolar diffusion occurring in many weakly-ionized plasmas because (i) it is limited to a plasma consisting of electrons and one type of positive ions, while weakly-ionized plasmas in air and many other gases have more than one type of positive ions and also contain various types of negative ions, and (ii) it is valid only for plasmas under no applied magnetic field. Ambipolar diffusion models for a four-component plasma (one type of positive ions, one type of negative ions, and electrons) were proposed by Thompson [7] and by Oskam [8]. The four-component ambipolar model by Thompson was further generalized by Suslov and Tirskii [9] and independently by Rogoff [10] and by Ramshaw and Chang [11,12] to plasmas with any number of ion species. While the latter models are general enough to be applicable to a multicomponent plasma, including those with negative ions, they cannot be applied to cases in which a significant induced or external magnetic field is present.

When under the influence of a magnetic field, the ambipolar diffusion in a three-component plasma (one type of positive ions, electrons, and neutrals) can be readily derived to be equal to (see for instance Refs. [13–15], and Ref. [1, p. 189]):

$$D_a = \left(1 + \frac{T_e}{T_i}\right) \frac{D_i}{1 + \mu_e \mu_i \boldsymbol{B}_\perp^2} \tag{2}$$

where $\boldsymbol{B}_\perp$ is the component of the magnetic field vector perpendicular to the diffusion velocity, μ_e is the electron mobility, and μ_i is the ion mobility. However, the latter cannot be applied to a plasma having negative ions or several types of positive ions with different mobilities. In Refs. [16–18], and more recently [19–21], several theories were proposed to overcome this limitation. Despite an in-depth theoretical treatment of the physical processes, the latter studies came short of yielding a general expression for the ion flow rate that is (i) applicable to a multicomponent magnetized plasma, and (ii) that can be used in the general case independently of the electric field, magnetic field, or current distribution.

Such a general expression for the ion flow rate was rather first proposed by Ramshaw and Chang [22] and dubbed the SCEBD model (Self-Consistent Effective Binary Diffusion Approximation). The SCEBD model is, however, limited to a two-temperature plasma in which all the ions share the same temperature as the neutrals, and also uses approximate weighting factors. To improve the model, Ramshaw and Chang subsequently recommended more accurate approximations to the weighting factors in Refs. [23,24]. Another substantial drawback of the SCEBD scheme is its lack of robustness at strong magnetic field. When the magnetic field is very strong, the scheme has been observed to be difficult to integrate and to result in erratic convergence behavior due to the strong coupling of the ambipolar diffusion terms with the electric and magnetic fields [25].

The objective of this paper is to develop a numerical method that would enable accurate modeling of ambipolar diffusion and drift phenomena in quasi-neutral plasmas in the general case (that is, in the presence of current, electric field, and magnetic field). Specifically, a newly recast expression for the ion flow rate is derived. This expression is generally valid for a multicomponent magnetoplasma and overcomes the shortcomings of SCEBD. Indeed, we will show that it is possible to recast the ion flow rate in terms of drift and ambipolar diffusion terms while not introducing approximate weighting factors or making other approximations. Furthermore, because the recast form of the ion flow rate proposed herein is not as strongly dependent on the electric field, the coupling with the potential equation is weakened. This results in a more robust numerical method that yields significantly faster convergence.

This paper is structured as follows. First, the physical model is presented. Second, we derive a recast form of the ion conservation equation that is applicable to a multicomponent plasma in the general case and that is amenable to integration. Third, several test cases are solved in 1D and 2D using the proposed method, and an assessment is made of the effect of a strong electric field and magnetic field on the performance of the method.

2. Physical model

While the recast form of the ion conservation equation that is presented herein is intended to be used in CFD codes in which the neutrals momentum, energy, and mass conservation equations are solved, the physical model is here limited for ease of reproducibility to the charged species conservation equations and the electric field potential equation, with the neutrals properties being set to a constant throughout the domain.

2.1. Charged species conservation equation

For a multicomponent plasma, it is necessary to solve a separate conservation equation for each type of charged species: