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An accurate and efficient method for treating aerodynamic interactions of cloud droplets

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ABSTRACT

Motivated by a need to improve the representation of short-range interaction forces in hybrid direct numerical simulation of interacting cloud droplets, an efficient method for treating the aerodynamic interaction of two spherical particles settling under gravity is developed. An effort is made to ensure the accuracy of our method for any inter-particle separation by considering three separation ranges. The first is the long-range interaction where a multipole method is applied. After a decomposition into six simple configurations, explicit formulae for drag forces and torques are derived from an approximate Force–Torque–Stresslet (FTS) formulation. The FTS formulation is found to be accurate when the separation distance normalized by the average radius is larger than 5. The second range concerns the short-range interaction where the interaction force could be very large. Leading-order lubrication expansions are employed for this range and are found to be accurate when the normalized separation is less than about 0.01. Finally, for the intermediate range where no simple method is available, a third-order polynomial fitting is proposed to bridge the treatments for long-range and short-range interactions. After optimizing the precise form of polynomial fitting and matching locations, the force representation is found to be highly accurate when compared with the exact solution for Stokes flows. Using this method, collision efficiencies of cloud droplets sedimenting under gravity have been calculated. It is shown that the results of collision efficiency are in excellent agreement with results based on the exact Stokes flow solution. Collision efficiency results are also compared to previous results to further illustrate the accuracy of our calculations. The effects of particle rotation and the attractive van der Waals force on the collision efficiency are also studied. The efficient force representation developed here is more general than the usual lubrication expansion and thus can serve as a better approach to correct unresolved short-range interactions in particle-resolved simulations.

1. Introduction

In recent years an increasing number of studies have been initiated to quantify the effects of air turbulence on the growth of cloud droplets during warm rain initiation, as it is believed that the in-cloud turbulence can enhance the rate of collision–coalescence and as such provides a mechanism to overcome the bottleneck between the diffusional growth and the

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gravitational collision–coalescence mechanism (see [1,2] and references therein). Cloud droplets of radius less than 10 μm grow efficiently through diffusion of water vapor, and droplets larger than 50 μm in radius grow efficiently through gravitational collisions [3]. Much recent attention has therefore been directed to the enhanced collision–coalescence rate by air turbulence for cloud droplets in the size range from 10 to 50 μm in radius. It has been shown that a moderate enhancement (*i.e.*, by a factor of two to three) of the collision kernel by air turbulence can significantly accelerate the growth of cloud droplets to form drizzle drops [2,4,5].

Air turbulence can enhance the collision–coalescence rate in two general ways. First, for the simplified problem of geometric collision neglecting aerodynamic interaction of cloud droplets, air turbulence can increase the collision rate by three possible mechanisms (see [2,4] and references therein): (1) enhanced relative motion due to differential acceleration and shear effects; (2) enhanced average pair density (*i.e.*, the number of interacting droplet pairs per unit volume) due to local preferential concentration of droplets; and (3) enhancement due to selective alterations of the settling rate by turbulence. Second, air turbulence can alter the collision efficiency of cloud droplets [6], namely, the ratio of the number of droplet pairs that can come into contact under the influence of local aerodynamic interaction, to the number of colliding droplet pairs without considering the local aerodynamic interaction. Air turbulence affects the collision efficiency by (1) enhancing the far-field relative motion of droplets and (2) introducing local flow shear and acceleration which modifies the aerodynamic interaction forces on droplets.

Compared to the geometric collision, collision efficiency is a more difficult problem as the disturbance flows introduce another set of length and time scales in addition to the background air turbulence. While there are quite a few studies in the literature concerning the collision efficiency of cloud droplets without air turbulence, there are very few studies devoted to the collision efficiency in a turbulent flow (e.g., see [6] and references therein). As pointed out in [7], previous studies often predicted different levels of enhancement of collision efficiency. This in part results from different kinematic formulations used to define the collision efficiency in different studies, some of which are not applicable to turbulent collisions. More importantly, there is currently a lack of accurate and consistent representations of aerodynamic interaction of many droplets in a turbulent flow.

As a first step in developing a better computational method for treating aerodynamic interaction of cloud droplets in a turbulent flow, an improved superposition method (ISM) was introduced in [8] to quantify the collision efficiency of cloud droplets in still air. The basic idea is to impose, in some average sense, the no-slip boundary condition on the surface of each droplet to better determine the magnitude and coupling of the Stokes disturbance flows in a many-droplet system. The no-slip boundary condition is specified either at the center of each droplet (the center-point formulation) or by an integral average over the droplet surface (the integral formulation). The advantage of ISM is that the application to many-droplet interactions in a turbulent airflow is rather straightforward leading to a hybrid direct numerical simulation (HDNS) approach [9,10]. The HDNS approach combines direct numerical simulation of the background air turbulence with an analytical representation of the disturbance flow introduced by many droplets. The approach takes advantage of the fact that the disturbance flow due to droplets is localized in space and there is a sufficient length-scale separation between the droplet size and the Kolmogorov scale of the background turbulent flow. This hybrid approach provides, for the first time, a consistent, quantitative tool for studying the combined effects of air turbulence and aerodynamic interactions on the motion and collisional interactions of cloud droplets. The disturbance flow is coupled with the background air turbulence through the approximate implementation of the no-slip boundary conditions on each droplet. Dynamical features in three dimensions and on spatial scales ranging from a few tens of centimeters down to 10 μm are captured. Both the near-field and the far-field droplet–droplet aerodynamic interactions could be incorporated [11].

HDNS provides a framework for a systematic improvement of the approach. In this regard, the HDNS approach is closely related to the multipole expansion method [12], also in general known as the *Stokesian dynamics* approach [13]. In fact, the center-point formulation of ISM is essentially the zero-moment expansion with only monopole terms and without the Faxen correction [14,15], while the integral formulation of ISM is the zero-moment expansion with the Faxen correction since the integral average of disturbance flow velocity over a droplet surface is equivalent to the center-point velocity plus the Faxen term. Here moments mean the force moments in the multipole expansion of Stokes flow solution around rigid particles [12]. Durlofsky et al. [12] presented a multipole formulation known as the Force–Torque–Stresslet (FTS) formulation which includes moments up to the first-order plus Faxen terms. This multipole expansion method considers many-body interaction with Stokes disturbance flows superimposed onto a nonuniform background flow.

The authors of [8,12] recognized that ISM and FTS cannot handle correctly short-range or lubrication forces. The short-range interaction forces, in principle, would require all higher-order moments to be included in the multipole expansion [16]; and the convergence to the exact lubrication forces is usually slow in the multipole expansion approach [17,16]. To accurately treat the lubrication force, Durlofsky et al. [12] made use of the exact force representation of the two-sphere problem (e.g., [17,18]) and at the same time, properly remove the redundant part from the multipole many-body representation. This procedure could be complicated for the many-droplet problem.

As a logical next step to the ISM, in this paper, we develop an efficient approach for two-droplet aerodynamic interaction in still air with accurate force representation for all separation distances. The results will be compared against the exact solutions of Jeffrey and Onishi [17] (Hereafter will be referred to as JO84). Our approach is to divide the problem into three subproblems. First, for long-range interactions, we apply FTS to six independent, simple configurations (see Fig. 1) which then in linear combination can be used to handle any long-range interaction of two unequal-size droplets. Second, the short-range interaction will be treated by a few leading order terms from the explicit lubrication expansion of JO84. Then guided by the