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A time-reversal lattice Boltzmann method

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ABSTRACT

In this paper we address the time-reversed simulation of viscous flows by the lattice Boltzmann method (LB). The theoretical derivation of the reversed LB from the Boltzmann equation is detailed, and the method implemented for weakly compressible flows using the D2Q9 scheme. The implementation of boundary conditions is also discussed. The accuracy and stability are illustrated by four test cases, namely the propagation of an acoustic wave in a medium at rest and in an uniform mean flow, the Taylor–Green vortex decay and the vortex pair–wall collision.

1. Introduction

Noise source identification is of major interest in the transports industry. The sound generated by an aerodynamic source is radiated in the flow in a one-way process: the source defines the sound field in the flow, but it is very difficult to identify the location of emission from the sound field. Noise source identification has been addressed by a large variety of methods. Among the three major families of methods, namely, those based on aeroacoustic analogies (see [1]), on statistical definition by correlation (see [2]) and on an inverse problem, we focus on the last one. The methods in this family solve an inverse propagation problem. After running a simulation for some time, the time is reversed and the simulation is run backwards. The study of inverse problems was at first used for antenna problems, where the reverse problem corresponds to a very simple wave-propagation model. The progresses in computational fluid dynamics allow more complex physical models to be solved (in our case the weakly compressible Navier–Stokes equations), and hence more accurate solutions of the inverse problem. The noise source detection is then performed using a sensitivity analysis, arguing that the higher the sensitivity of the acoustic field to a hydrodynamic event is, the more likely it is to be its source. The sensitivity analysis can be done with an adjoint problem [3] or with complex differentiation [4], this specific topic is left for a future work. Here, we focus on the resolution of the reverse hydrodynamic problem.

In the past two decades, the lattice Boltzmann method (see, e.g. the book by Succi [5], Benzi et al. [6] or Aidun and Clausen [7]) has gained fame amongst the computational fluid dynamics community. The method is well suited for parallel implementation, like demonstrated in [8,9] and numerous recent parallel codes implement it [10–12]. Its efficiency compared to other numerical methods for CFD at moderate Mach numbers has been demonstrated for example by Geller et al. [13]. The LBM is a good candidate for the simulation of weakly compressible flows, and is, therefore, a good candidate for the numerical resolution of aeroacoustic problems (see [14,15]).

As it is governed by the Euler equations, the propagation of an acoustic wave in a perfect inviscid fluid is an isentropic and reversible phenomenon. The inverse of the forward in time equations are exactly the same as those with reversed time. Then, from a given sound field in a given flow, it should be possible to rewind time and focus on the sound source. In the acoustics domain, this method is widely used, for example by Fink et al. [16] who, by recording the sound waves on the boundary of a

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volume, and emitting the time-reversed sound waves, focuses on the source inside the domain. Applications to this method range from medical imaging and therapy to non-destructive control. The method has been extended to the Euler equations by Deneuve et al. [4]. In their work, the Euler equations are solved using a pseudo-characteristic formulation, and the quantities at the boundary of the domain are stored. At the end of the simulation, the final state and the recorded data on the boundaries are time-reversed and the reversed Euler problem is solved using the same numerical method.

In the present paper we propose to use the lattice Boltzmann method to solve the weakly compressible Navier–Stokes equations and investigate the resolution of time-reversed problems with the LBM. We will show that the time-reversed lattice Boltzmann equation leads asymptotically to the time-reversed Euler and Navier–Stokes equations. These theoretical results are then validated with four two dimensional benchmarks: the propagation of acoustic waves in zero or in subsonic uniform mean flows, the Taylor–Green vortex decay and the vortex pair–wall collision. When dealing with dissipative media, the time symmetry of the acoustics equations is broken, but the time reversal procedure still holds. Serrin [17] initiated a long series of publications about the recovery of initial values for the Navier–Stokes equations. More contributions are available in [18–20], who concluded that time-reversed Navier–Stokes equations are well-posed over finite time and that their solution depend continuously on initial data. Thanks to the fact that our LBM method is equivalent to weakly compressible isothermal Navier–Stokes equations, it seems legitimate to at least try to compute the solution of time-reversed LBM equations over finite times and the algorithm is numerically unstable. We show that for time t_1 small enough, the algorithm is capable of recovering the data for $t_1 \leqslant t \leqslant 0$. Laboratory experiments by Griffa et al. [21], addressed the time-reversal approach for wave equation in dissipative media. He concluded that "*in the case of attenuative media the symmetry property of the wave equation is no longer valid. However, the TRP still holds but with decreased efficiency: part of the spatial frequencies associated with the forward propagating wave fields never reach the TRM due to dissipation. The back-propagated wave fields still retro-focus on the position(s) of the source(s) and/or point-like scatterer(s) from the forward propagation*". The experience shows that despite dissipation, one is still able to retro-focus waves on their source. If the dissipation alter the precision of the initial data recovery, it will yield a stable algorithm that should be used to simulate time backward wave propagation with the lattice Boltzmann method.

The paper is organised as follows. In Section 2 the time-reversion of the LBGK equation is exposed and the equivalence with the time-reversed Euler and Navier–Stokes equations is proved. The LB algorithm with minimal error for the reverse simulation, as well as the boundary conditions, are detailed in Section 3. In Section 4 the numerical examples are presented, from the propagation of an acoustic wave, in a closed or open cavity, to flows with high viscous effects. Finally, this paper is concluded in Section 5 and perspectives are given.

2. The time-reversed Boltzmann equation and its macroscopic limits

In this section we discuss the time-reversed Boltzmann equation and show its equivalence with the time-reversed Euler and Navier–Stokes equations.

2.1. Time-reversed Boltzmann–BGK equation

The Boltzmann equation with the BGK collision operator governs the evolution of the density probability function $f(\boldsymbol{x},\xi,t)$ of finding a particle at position $\boldsymbol{x}$ at time t with velocity ξ. It reads in the absence of an external force (see [22]):

$$\frac{\partial f}{\partial t} + \xi \cdot \nabla f = -\frac{1}{\tau}(f - f^{eq}),$$

where τ is the relaxation time of the fluid, and f^{eq} the Maxwellian equilibrium distribution. If $f(\boldsymbol{x},\xi,t)$ is a solution of the Boltzmann BGK (FBBGK, for forward Boltzmann BGK) equation, $\tilde{f} = f(\boldsymbol{x}, -\xi, -t)$ is a solution of the time reversed equation, $t \to -t$ and $\xi \to -\xi$, that will be called reversed Boltzmann BGK (RBBGK):

$$\frac{\partial \tilde{f}}{\partial t} + \xi \cdot \nabla \tilde{f} = -\frac{1}{\tilde{\tau}}\left(\tilde{f} - \tilde{f}^{eq}\right), \tag{1}$$

with $\tilde{f}^{eq} = f^{eq}(\boldsymbol{x}, -\xi, -t)$. We draw the attention of the reader that the RBBGK equation is very similar to the FBBGK, except for the fact that the relaxation time τ is replaced by $\tilde{\tau} = -\tau$ (the reverse relaxation time, $\tilde{\tau}$, becomes negative). The macroscopic moments of the distribution function are defined as:

$$\tilde{\rho} = \int \tilde{f} \mathrm{d}\xi,$$
$$\tilde{\boldsymbol{\jmath}} = \tilde{\rho}\tilde{\boldsymbol{u}} = \int \xi \tilde{f} \mathrm{d}\xi,$$
$$\widetilde{\boldsymbol{P}} = \int \xi\xi \tilde{f} \mathrm{d}\xi - \tilde{\rho}\tilde{\boldsymbol{u}}\tilde{\boldsymbol{u}},$$

where $\tilde{\rho}, \tilde{\boldsymbol{\jmath}}, \tilde{\boldsymbol{u}}$ and $\widetilde{\boldsymbol{P}}$ are respectively the time reversed density, momentum, velocity and pressure (or stress) tensor, and where $\xi\xi$ stands for the tensor product of ξ with ξ.