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A new formulation of Kapila's five-equation model for compressible two-fluid flow, and its numerical treatment

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ABSTRACT

A new formulation of Kapila's five-equation model for inviscid, non-heat-conducting, compressible two-fluid flow is derived, together with an appropriate numerical method. The new formulation uses flow equations based on conservation laws and exchange laws only. The two fluids exchange momentum and energy, for which exchange terms are derived from physical laws. All equations are written as a single system of equations in integral form. No equation is used to describe the topology of the two-fluid flow. Relations for the Riemann invariants of the governing equations are derived, and used in the construction of an Osher-type approximate Riemann solver. A consistent finite-volume discretization of the exchange terms is proposed. The exchange terms have distinct contributions in the cell interior and at the cell faces. For the exchange-term evaluation at the cell faces, the same Riemann solver as used for the flux evaluation is exploited. Numerical results are presented for two-fluid shock-tube and shock-bubble-interaction problems, the former also for a two-fluid mixture case. All results show good resemblance with reference results.

1. Introduction

The research interest in modeling and computing compressible, two-fluid flows is longstanding, as is reflected from the 26-year old review article by Stewart and Wendroff [24], which already contains a wealth of literature on the topic.

These days, to model two-fluid flows, seven-equation models are the most complete. Baer and Nunziato's [4] is the best known model in this class. For both fluids, it contains equations for the quantities mass, momentum and energy, already implying six equations. The seventh equation describes the topology of the flow, e.g., the location and shape of the two-fluid interface. A more recent seven-equation model has been proposed by Romenski et al. [19]. Important modeling and numerical work on seven-equation models has been done by Saurel and Abgrall [20].

Besides completeness, a seven-equation model also implies complexity, both physically and numerically. Since its general physics is not always necessary, simpler and more compact models have been proposed and successfully applied. An elegant hierarchy of reduced models exists, with the numbers of equations ranging from six to three only. Examples of the latter are the homogeneous equilibrium model [5] and the barotropic model described in [26]. For a clear and compact overview over existing reduced two-fluid flow models, we refer to [6].

An important class of reduced models is formed by the five-equation models, in which velocity equilibrium and pressure equilibrium are considered, due to zero relaxation time. Both equilibria are valid across two-fluid interfaces modeled as a

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contact discontinuity, a practically relevant case. The archetype five-equation model is that of Kapila et al. [10]. It has already found many applications, a non-exhaustive list of excellent references is [12,3,14,2,21,17]. (Recently, Saurel et al. have even derived a six-equation model from a five-equation one [22].) Kapila's five-equation model contains four equations for conservative quantities: two for mass (e.g., bulk mass and mass of one of the two fluids), one for bulk momentum, and one for bulk energy. Like the Baer–Nunziato model, it is also completed by an equation for a non-conservative quantity describing the flow topology, in the Kapila model, a non-homogeneous convection equation for the volume fraction of one of the two fluids.

In the present paper, we derive a new formulation of Kapila's five-equation model. The first four equations are the same as in Kapila's five-equation model, our fifth equation, which substitutes the topological equation from Kapila's formulation, is new. It is the energy equation for one of the two fluids. In [22], the energy equation for one of the two fluids is also considered, though in a six-equation context. The advantage of the current new five-equation formulation is that all equations can now be written in integral form, in a single system. Our fifth equation contains a righthand side that describes the exchange of energy between the two fluids, in terms of rate of work. Whereas the first four equations are conservation laws, the fifth equation is an exchange law; it is conservation-law-like though. As a consequence, the entire system allows for a rather straightforward derivation and application of finite-volume methods and corresponding numerical tools, such as approximate Riemann solvers. In the current manuscript, an Osher-type approximate Riemann solver is derived for the new five-equation formulation. For the numerical treatment of the righthand side in the fifth equation, the energy-exchange terms, use is also made of the just derived approximate Riemann solver.

In differential form, the present formulation is actually identical to Kapila's. However, in the form in which it is derived, discretized and solved, the predominant integral form and corresponding finite-volume method, it is different; its fifth equation is new and consistent with the other four equations.

The contents of the paper is the following. In Section 2, the new five-equation formulation is derived. Most attention is for the derivation and physical interpretation of the energy-exchange terms. In Section 3, the numerical method for the five-equation model is presented, with – also here – most attention for the energy-exchange terms. In Section 4, numerical results are presented, for three shock-tube problems and two standard shock-bubble-interaction problems. Section 5 concludes the paper.

2. New five-equation formulation

2.1. Assumptions

Each of the two fluids is assumed to be mass conservative. They may exchange momentum by exerting forces on each other, and may exchange energy due to work. The amount of momentum and energy exchange depends on the relaxation speed of pressure and velocity. An important assumption is instantaneous relaxation of the pressure and the velocity vector, i.e.,

$$p_1 = p_2 \equiv p, \tag{1a}$$
$$\boldsymbol{u}_1 = \boldsymbol{u}_2 \equiv \boldsymbol{u}, \tag{1b}$$

where the subscripts refer to the two fluids. Assumption (1) prescribes that the pressures and velocity vectors on both sides of a two-fluid interface are equal. It is the known step to reduce a seven-equation model to a five-equation model, see, e.g., [1,20,10,3,14,6]. Further, viscosity and heat conduction are neglected. Since this implies that the two fluids do not exchange heat, in general, there will be no thermal equilibrium.

2.2. Conservation and exchange laws

Consider a control volume V, which is fixed in space and time. For this volume three physical principles are known: conservation of bulk mass, bulk momentum and bulk energy, in integral form written as:

$$\frac{\partial}{\partial t}\int_V \rho\,\mathrm{d}V + \oint_S \rho\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S = 0, \tag{2}$$
$$\frac{\partial}{\partial t}\int_V \rho\boldsymbol{u}\,\mathrm{d}V + \oint_S \rho\boldsymbol{u}\otimes\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S + \oint_S p\boldsymbol{n}\,\mathrm{d}S = \boldsymbol{0}, \tag{3}$$
$$\frac{\partial}{\partial t}\int_V \rho E\,\mathrm{d}V + \oint_S \rho E\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S + \oint_S p\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}S = 0. \tag{4}$$

There can be two fluids in the control volume V. The pressure and velocity are equal over a two-fluid interface, but the density and total energy do not need to be so; in general $\rho_1 \neq \rho_2$ and $E_1 \neq E_2$. The quantities ρ and E in Eqs. (2) and (3) are the bulk density and bulk total energy.

For each fluid separately an equation for mass, momentum and energy can also be written. The mass of each fluid is conserved. The conservation of, e.g., mass of fluid 1 can be written as