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Interlocked optimization and fast gradient algorithm for a seismic inverse problem

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ABSTRACT

We give a nonlinear inverse method for seismic data recorded in a well from sources at several offsets from the borehole in a 2D acoustic framework. Given the velocity field, approximate values of the impedance are recovered. This is a 2D extension of the 1D inversion of vertical seismic profiles [18]. The inverse problem generates a large scale undetermined ill-conditioned problem. Appropriate regularization terms render the problem well-determined. An interlocked optimization algorithm yields an efficient preconditioning. A gradient algorithm based on the adjoint state method and domain decomposition gives a fast parallel numerical method. For a realistic test case, convergence is attained in an acceptable time with 128 processors.

1. Introduction

Recovering structural and quantitative information about the crust composition is a fundamental problem in industrial applications including seismic prospecting and CO_2 storage monitoring. Since direct measurements only give very local information, seismic imaging methods are based upon indirect measures of the subsurface parameters obtained with measurements of acoustic probes. One measures the subsurface response to an impulse at the ground surface. The impulse is generated by seismic sources called the illuminating wave. This source field propagates through the subsurface and undergoes reflection, refraction, diffraction, transmission and attenuation. These interactions between waves and subsurface are governed by the subsurface physical properties. Traditionally, receivers located hundreds of meters from the sources record the reflected signal at the ground surface. The recordings constitute the seismic data. Seismic imaging inverse methods process this data in order to estimate the geophysical subsurface parameters. Given a seismic wave propagation model, the estimated parameters must reproduce as accurately as possible the data that have been recorded. Typical lengths of investigation can reach several kilometers in depth and in lateral directions.

Such seismic imaging methods have been intensively used and improved during the last three decades. The most frequently used ones are "linear" inverse methods. Denote by m the subsurface parameters that have to be estimated. The wave propagation model is linearized around a smooth approximation m_0 of m, and an estimation of a perturbation dm is sought, such that $m \approx m_0 + dm$. This linearization relies on the assumption that the part of the recorded signal that depends nonlinearly on dm can be neglected. Given a prior estimation of the seismic source, these methods result in a linear least squares problem which can be efficiently solved. They provide quantitative estimates of the subsurface parameters with a spatial resolution (that is to say the smallest length scale resolved) reaching about 25 m [6].

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These linear inverse methods are faced with two important difficulties:

- determining a sufficiently accurate background estimation m_0 can be difficult or impossible;
- for strongly heterogeneous media, the part of the seismic signal depending nonlinearly on dm increases, rendering the linearization inappropriate.

In these two cases, the wave propagation model is not likely to give a good interpretation of the seismic data. As a result, the approximation for dm is not satisfactory [9,10].

This leads one to try to solve the inverse problem without the linearization approximation. There is evidence that this can alleviate these two difficulties, and even increase the resolution of the resulting method [11,12]. Unfortunately, in this case, another difficulty arises, related to the accuracy of the estimation of the seismic source. While the linearized inverse problem is not sensitive to an error on the source estimation, the nonlinear inverse problem is. This results in the impossibility of inverting ground surface seismic data with the nonlinear method.

Another type of imaging method, called *vertical seismic profile inversion*, was introduced in the 80s [18]. For this method, receivers are placed in a vertical well instead of being located at the ground surface. This configuration provides an interesting advantage. When receivers are located at the ground surface, they record only reflected signals. When they are located in a well below the sources, the receivers record both direct and reflected signals. The resulting data contain non negligible information from the illuminating wavefield that has directly passed through the substrate. This method can simultaneously recover the illuminating wavefield and the subsurface parameters from the data. The illuminating wavefield thus becomes an unknown. This opens the door to attacking the fully nonlinear problem.

Such a method was first implemented by Macé and Lailly in a 1D aoustic wave propagation problem [18]. The subsurface is described by two parameters: the acoustic impedance $I(z)$ and the seismic wave velocity $c(z)$. The latter is first estimated from arrival times. The method then recovers approximate acoustic impedance values. For the 1D problem, the shallowest receiver records a signal $h(t)$ corresponding to a Dirichlet boundary condition for a 1D wave propagation model starting at the depth z_1 of the first receiver. The method consists in finding the acoustic impedance model $I(z)$ from the depth z_1 and the Dirichlet condition $h(t)$ that minimizes the misfit between the data and the model prediction. The problem is proved to be well-posed, and the impedance model is uniquely determined up to a multiplicative constant [3,4]. The method is stable and robust even with highly noise-corrupted data. In addition, the impedance estimation shows an excellent vertical resolution, which suggests the interest of solving the fully nonlinear problem.

However, the 1D propagation assumption is not realistic, and the acoustic impedance estimation in only valid in the vicinity of the well. Therefore, extending this 1D nonlinear seismic imaging method to a multidimensional framework is an attractive idea. Such an extension could lead to the estimation of subsurface parameters within a zone around the well. The present paper proposes an implementation of such an extension. A 2D acoustic wave propagation problem is investigated. As in 1D, the subsurface is described by the wave velocity and the acoustic impedance. On the basis of an assumed known velocity field, the method aims at recovering the 2D acoustic impedance.[2] Walkaway data is used. Seismic sources are placed at different offsets from the borehole in order to illuminate horizontal variations of the medium. For each location of the source, the particle displacement velocities are recorded by an array of receivers in a well. This extension to a 2D framework of the originally 1D method raises two main difficulties.

- The 2D inverse problem is undetermined and requires additional *a priori* information on the unknowns.
- For realistic applications, the 2D inverse problem results in a large scale ill-conditioned nonlinear minimization problem for which an appropriate numerical method has to be constructed.

In [21], we mainly focused on the first of these two problems and proposed suitable regularization techniques. Here we focus on the second problem. The numerical method involves an interlocked optimization algorithm based upon an external quasi-Newton loop used to estimate the impedance, together with a conjugate gradient inner loop, used for the computation of the Dirichlet condition. The algorithm requires a fast determination of the gradient of the misfit function. The adjoint state approach is used together with a domain decomposition method to yield a parallelizable algorithm. The efficiency of this numerical method is evaluated on a realistic test case, with synthetic data. The results are encouraging.

- They show satisfactory convergence of the method in a reasonable time.
- The 2D acoustic impedance distribution is estimated with a high resolution within a 200 m and 3 km depth rectangle around the well.

The article is organised as follows. In Section 1, the 2D inverse problem is described in detail. In Section 2, the numerical method used to solve this inverse problem is presented. In Section 3 the results of the test case are exposed.

[2] This assumption is important. The determination of the wave velocity field from seismic data is a fundamental problem with its own methods [28]. We assume an approximation for $c(x)$ is available and focus on determining $I(x)$.