Journal of Computational Physics 248 (2013) 147–176

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Three-dimensional simulation of a solid–liquid flow by the DEM–SPH method

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ARTICLE INFO

Article history:
Received 19 October 2012
Received in revised form 11 February 2013
Accepted 7 April 2013
Available online 25 April 2013

Keywords:
Solid–liquid flow
Two-phase flow
Fluid–particle interaction
Discrete element method
Smoothed particle hydrodynamics
Wall boundary conditions

ABSTRACT

In this paper, we describe a new Lagrangian–Lagrangian algorithm, which is referred to be the DEM–SPH method, for solid–liquid flows involving free surfaces. The DEM solid phase and the SPH liquid phase are coupled using the local averaging technique described by Lagrangian approaches, where both the continuity equation and the interaction force, i.e. drag force, are connected with the local mean voidage. Conservative forms of momentum transformation are derived for the DEM–SPH interaction via a variational approach. By introducing a correction to the SPH approximation with explicit inclusion of boundary information, arbitrary boundaries can be modeled without any extra wall particles, where the boundary is used commonly for both DEM and SPH phases. We deploy level-set distance functions to efficiently construct and evaluate this boundary model. To examine the validity of the present method, we perform three-dimensional simulations of a dynamic flow in a solid–liquid dam break and a quasi-steady flow in a rotating cylindrical tank; and we conduct validation experiments to justify the simulation results. In the dam-break problem, positions of wave fronts during the collapse are computed and compared with experimental measurements; for the circulating tank, some macroscopic aspects of the steady flow, e.g. the shape, dimension and velocity profile of the solid bed, are obtained for validation data. In both cases, the simulation results are in good agreement with those of the experiment. Consequently, the DEM–SPH method is proved to be adequate in modeling solid–liquid flows through this study.

1. Introduction

Solid–liquid flows are encountered in many applications such as sedimentation, slurry transport and rheology. The solid–liquid flows are widely studied in science and engineering. Accurate numerical simulations would be helpful both for understanding the complex phenomena and optimizing the design and investigation of operational conditions in these processes. For these simulations, modeling of the moving boundary, a free surface fluid flow and solid–liquid coupling become important, where a Lagrangian method can be promising. In comparison with some traditional Eulerian formulations, Lagrangian schemes may be effective for modeling the moving boundaries and free surface flows. This is because the Lagrangian approaches usually deal with discrete elements and do not need complicated techniques like mesh generation or adaptive mesh.

As far as the Lagrangian approaches for fluid flows are concerned, some truly mesh-free methods, i.e. smoothed particle hydrodynamics (SPH) [1] and the moving particle semi-implicit method (or moving particle simulation) (MPS) [2,3], have

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http://dx.doi.org/10.1016/j.jcp.2013.04.019

been thoroughly studied thus far. These Lagrangian methods have some advantages, in that they use straightforward convection models which are not affected by the so-called numerical diffusion, and can simulate large deformation, fragmentation and folding of the free surface flow relatively easily. These methods can also simulate moving objects in a fluid flow with ease. Therefore, these Lagrangian approaches have been applied across a broad range of engineering disciplines to compute various environmental or industrial fluid flows, for example, in marine [4,5], coastal [6–9], civil engineering [10], casting [11], nuclear engineering [12] and membrane phenomenon [13]. As for simulating discontinuous systems consisting of granular matters and powders, the discrete element method (DEM) [14] is widely recognized for its simplicity and capability. The DEM is also a Lagrangian method simulating the behavior of individual solid particles based on Newton's second law of motion. In the DEM, the contact force is evaluated using the Voigt model consisting of springs, dashpots and a friction slider. The DEM has been applied to various powder systems, e.g. fluidized beds [15–17], ball mills [18,19], and pneumatic conveying [20].

When it comes to the application of these Lagrangian methods to solid–fluid flows, two types of approaches have been employed so far. One is the direct numerical simulation (DNS) and the other is the local averaging technique [21]. In DNS-based methods, the hydrodynamics forces acting on a solid are evaluated by solving the Navier–Stokes equation directly. The disadvantage is that extremely fine resolution (i.e. the number of the SPH or MPS particles) is required to reproduce the fluid motion precisely [5,6,22]. As a result, the DNS-based method is difficult to be applied to large-scale practical systems, though the hydrodynamics force can be calculated accurately without any experimental equations. On the other hand, the local volume average technique may be able to be used to calculate a multiphase flow in a large-scale system. The governing equations transformed by the local volume average technique have often been employed in the modeling of practical-scale systems, e.g. gas–solid flows such as fluidized beds. Although empirical equations are needed to calculate the drag force, the number of computation points becomes significantly smaller than that of the DNS-based method. A Lagrangian–Lagrangian approach was developed by coupling the DEM with the MPS method, where the effects of hydrodynamics forces and physical properties on macroscopic behavior of solid particle beds were investigated in a two-dimensional system [23]. It is illustrated that the other hydrodynamics forces except for the drag force were negligible in this system, whilst relaxed contact forces could be applied by softer solid particles from the viewpoint of engineering.

Similar to the MPS method, the SPH method is also featured as a Lagrangian approach that is truly mesh-free. The SPH method has been applied to multi-component flows. For multi-fluid flows, a variety of SPH works have been proposed to capture the discontinuity and model fluids motion [24,25]. Inception of applying SPH method to multiphase flows with interpenetrating particles could be dated back to some early literatures: a two-fluid model [21,26] was resolved in SPH [27] and applied to a blend of gas and dust [28], while recently an alternative attempt was to simulate the gas–solid bubbling using SPH [29]. In the past study, the SPH method was coupled with the DEM, where the volume average technique was introduced into the liquid phase [30]. However, this approach seemed to have some problems: not showing the variational consistency in the underlying SPH system, traditional SPH formulations were fixed by local averaging directly in their work (in fact, in [30] the SPH form used for the continuity equation was less compatible in the sense of weak formulations, e.g. see [31] for a discussion of the SPH continuity equations); the wall boundary condition was treated in a spring-dashpot algorithm such as the DEM, which was not a fluid dynamical statement at all; also it was suspected that some fundamental principles, both the momentum balance according to Newton's third law of motion and the fluid displacement caused by immersed objects, were not modeled precisely, since the net force of interaction terms on a single particle were not represented properly in summations of pairwise relations. Furthermore, modeling of the wall boundary condition in arbitrary shapes remained to be an arguable aspect asking for further research in the SPH method. Consequently, adequacy of the modeling of solid–liquid flows has not been proved by using the DEM and SPH method.

To handle those problems, in the present study, we carry out a new DEM–SPH algorithm, where DEM and SPH phases are coupled using the local averaging technique. We present a novel variational formulation of the pressure-based interphase interaction to ensure conservative properties. Suffice to say other fundamental aspects, both the reciprocal principle of Newton's third law of motion and the displacement of immersed bodies, are modeled properly. Besides, an implementation of wall boundaries using level set functions is brought in for practical simulations. As a numerical example, the three-dimensional simulation of a solid–liquid dam-break flow is performed, through which the evolutions of the wave front are evaluated and compared with those gained from a validation test. Another numerical study is the three-dimensional simulation of a quasi-steady solid–liquid flow in a cylindrical tank. Correspondingly a validation experiment is conducted as well. For both test cases, the simulation results are shown in good agreements with validating experiments. Consequently, the DEM–SPH method is shown to be adequate through this study.

2. Physical model and governing equations

As a place of departure, we will first review the elementary concept of local mean values and the local averaging technique [21] for the mechanics of solid–liquid flows in this section. The work by Anderson and Jackson [21] established a mechanical description, namely the two-fluid model, for the dynamics of particle–fluid fluidizations. With the introduction of local mean variables, it was able to transform the equations of motions for solid and liquid phases to represent inter-phase momentum exchange and balance.