Journal of Computational Physics 230 (2011) 6399–6403

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Short note

On the similarity between Dirichlet–Neumann with interface artificial compressibility and Robin–Neumann schemes for the solution of fluid-structure interaction problems

Joris Degroote *

Department of Flow, Heat and Combustion Mechanics, Ghent University, Sint-Pietersnieuwstraat 41, B-9000 Ghent, Belgium

ARTICLE INFO

Article history:
Received 28 November 2010
Received in revised form 11 April 2011
Accepted 16 May 2011
Available online 23 May 2011

Keywords:
Fluid-structure interaction
Partitioned
Robin–Neumann
Dirichlet–Neumann
Interface artificial compressibility
Finite volume

1. Introduction

In this note, the similarity between two implicit, partitioned solution techniques for fluid-structure interaction (FSI) problems is analyzed using finite volume discretization of the flow equations. Fluid-structure interaction refers to the mutual interaction between a fluid flow and a flexible structure. Partitioned solution techniques solve the flow equations and the structural equations separately. These techniques are classified as implicit (or strongly coupled) if they satisfy the interaction conditions on the fluid-structure interface in each time step and as explicit (or loosely coupled) if they do not.

Both techniques analyzed in this note use block Gauss–Seidel (GS) iterations, meaning that the flow equations and the structural equations are solved consecutively within a time step until some convergence tolerance is reached. As the flow and structural equations are solved separately, the interaction conditions on the fluid-structure interface have to be converted into boundary conditions on the common boundary of the fluid and structure subdomains. Several types of boundary conditions can be applied, resulting in different decompositions. In the case of Dirichlet–Neumann (DN) decomposition, the flow equations are solved with a Dirichlet boundary condition (given velocity) on the fluid-structure interface, while the structural equations are solved with a Neumann boundary condition (given stress) on the interface. Conversely, Robin–Neumann (RN) decomposition, introduced in [1], denotes a Robin boundary condition on the fluid side of the interface and a Neumann boundary condition on the structure side.

* Corresponding author.
E-mail address: Joris.Degroote@UGent.be.
URL: http://www.fsi.ugent.be/.

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2011.05.012

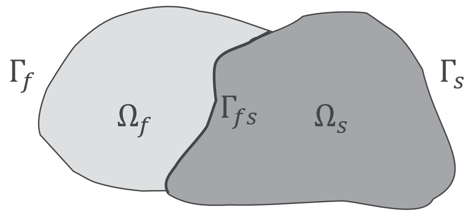

Fig. 1. The fluid subdomain Ω_f, the solid subdomain Ω_s, their boundaries Γ_f and Γ_s and the fluid-structure interface Γ_{fs}.

The first technique in this comparison is block Gauss-Seidel iterations applied to the monolithic system previously multiplied by a suitable permutation matrix, leading to a Robin–Neumann decomposition (GS–RN). This first technique includes a simplified version of the structural model in the flow equations by means of a Robin boundary condition to accelerate the convergence of the GS iterations [1,2]. The second technique is block Gauss-Seidel iterations with Dirichlet–Neumann decomposition and interface artificial compressibility (GS–DN–IAC). This second technique includes a local, linearized version of the structural model in the flow equations by means of pressure-dependent source terms in the fluid cells adjacent to the fluid-structure interface. Source terms were added to both the continuity equation and the momentum equations in [3], whereas only the continuity equation was modified in [4–6]. It is important to mention that the density is constant in the IAC method, despite the name of this method which was given due to the similarity with artificial compressibility schemes to solve flow problems [7]. In [8], Artificial compressibility was applied to the entire fluid domain and not only in the cells adjacent to the fluid-structure interface.

2. Governing equations

The fluid (f) and structure (s) subdomains are indicated as Ω_f and Ω_s and their boundaries as Γ_f and Γ_s. The fluid-structure interface $\Gamma_{fs} = \Gamma_f \cap \Gamma_s$ is the common boundary of these subdomains, as indicated in Fig. 1. The governing equations are immediately given in time-discrete form, using backward Euler discretization for simplicity. The notation δ_t is defined as

$$\delta_t z^{n+1} = \frac{z^{n+1} - z^n}{\Delta t}, \tag{1}$$

for any time-dependent variable z, with the superscript n denoting the time step and Δt the time step size.

The flow equations for the incompressible fluid with density ρ_f in Ω_f^{n+1} are given by

$$\nabla \cdot \boldsymbol{v}^{n+1} = 0, \tag{2a}$$

$$\delta_t \boldsymbol{v}^{n+1} + \nabla \cdot \boldsymbol{v}^{n+1}\left(\boldsymbol{v}^{n+1} - \boldsymbol{w}^{n+1}\right) - \frac{1}{\rho_f} \nabla \cdot \boldsymbol{T}_f^{n+1} = 0, \tag{2b}$$

in arbitrary Lagrangian–Eulerian (ALE) formulation, with $\boldsymbol{v}$ the fluid velocity and $\boldsymbol{w}$ the grid velocity. The interface position is treated implicitly. Body forces are omitted for simplicity. For a Newtonian fluid with dynamic viscosity μ, the Cauchy stress tensor $\boldsymbol{T}_f$ is defined as

$$\boldsymbol{T}_f = -p\boldsymbol{I} + 2\mu\boldsymbol{G} \tag{3}$$

with p the pressure and

$$\boldsymbol{G} = \frac{1}{2}(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\mathrm{T}}), \tag{4}$$

the strain rate tensor. The structure in Ω_s^{n+1} is governed by

$$\rho_s \delta_{tt} \boldsymbol{u}^{n+1} - \nabla \cdot \boldsymbol{T}_s^{n+1} = 0, \tag{5}$$

in Lagrangian formulation, with $\boldsymbol{u}$ the displacement. The relation between the Cauchy stress tensor $\boldsymbol{T}_s$ and the strain tensor is given by the constitutive law of the material. On the interface Γ_{fs}^{n+1}, the kinematic equilibrium

$$\boldsymbol{v}^{n+1} = \delta_t \boldsymbol{u}^{n+1} \tag{6a}$$

and the dynamic equilibrium

$$\boldsymbol{T}_f^{n+1} \cdot \boldsymbol{n}^{n+1} = \boldsymbol{T}_s^{n+1} \cdot \boldsymbol{n}^{n+1}, \tag{6b}$$

need to be satisfied, with $\boldsymbol{n}^{n+1}$ the normal pointing outwards of Ω_f^{n+1}. Moreover, the normal velocity of the fluid grid has to match the normal structural velocity on Γ_{fs}^{n+1}.