Journal of Computational Physics 230 (2011) 6688–6708

Contents lists available at ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Topology optimization of unsteady incompressible Navier–Stokes flows

Yongbo Deng [a,b], Zhenyu Liu [a,*], Ping Zhang [a], Yongshun Liu [a,b], Yihui Wu [a]

[a] State Key Laboratory of Applied Optics, Changchun Institute of Optics, Fine Mechanics and Physics (CIOMP), Chinese Academy of Sciences, 130033 Changchun, Jilin, China
[b] Graduate University of Chinese Academy of Sciences, Beijing, China

ARTICLE INFO

Article history:
Received 15 November 2010
Received in revised form 29 April 2011
Accepted 4 May 2011
Available online 10 May 2011

Keywords:
Topology optimization
Unsteady flow
Navier–Stokes equations
Continuous adjoint method

ABSTRACT

This paper discusses the topology optimization of unsteady incompressible Navier–Stokes flows. An optimization problem is formulated by adding the artificial Darcy frictional force into the incompressible Navier–Stokes equations. The optimization procedure is implemented using the continuous adjoint method and the finite element method. The effects of dynamic inflow, Reynolds number and target flux on specified boundaries for the optimal topology of unsteady Navier–Stokes flows are presented. Numerical examples demonstrate the feasibility and necessity of this topology optimization method for unsteady Navier–Stokes flows.

1. Introduction

Layout optimization of fluidic devices is an interesting field with respect to theory and application. The goal of optimization is to achieve better performance for a user specified objective that is related to certain characteristics of fluidic problems. Usually, layout optimization is categorized into shape optimization and topology optimization. Shape optimization improves the performance of a fluidic device by adjusting the positions of structural boundaries, keeping the topology of the structure unchanged. Topology optimization can optimize the shape and the topology of structures simultaneously. Therefore, topology optimization is a more general optimization technique than shape optimization. Currently, the density type method [1,2] and level set method [3–6] have been developed for implementing of topology optimization. Topology optimization by the density method was first used to design stiffness and compliance mechanisms [7–10] and has been extended to multiple physical problems, such as acoustic, electromagnetic, fluidic, optical and thermal problems [11–17]. Topology optimization by the density method for fluidic problems was first researched for Stokes flows [12,18,19] and Darcy–Stokes flows [20,21]. It was later extended to Navier–Stokes flows [22–26] and non-Newtonian flows [27]. Additionally, the topology optimization method has been applied to design fluidic devices [28–31]. The level set method, pioneered by Osher and Sethian [32], accomplishes the change of topology by evolving and merging the zero contour of the level set function. This method provides a general way to track the implicit interface between two phases, and it has been applied to fluidic shape and topology optimization [25,26]. One of the major advantages of the level set method lies in expressing continuously moving interfaces and abstracting the material domains that correspond to the structural topology. Recently, it has been observed that the conventional level set method may be inadequate for the cases in which the initial shape of the structure has fewer holes than the optimal geometry [4], especially in two-dimensional cases. The above difficulty can be

* Corresponding author.
E-mail address: liuzy@ciomp.ac.cn (Z. Liu).

0021-9991/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcp.2011.05.004

overcome using topological sensitivity, which was introduced by Sokolowski and Zochowski [33] for linear elasticity and that has been extended to several other linear and nonlinear physical problems [34–37]. In particular, the topological sensitivity has been researched for steady Stokes flows [38–41] and Navier–Stokes flows [42]. To the best of the authors' knowledge, there is no formally published paper discussing the topological sensitivity for unsteady Navier–Stokes flows. Therefore, the application of the level set method to topology optimization of the unsteady flows is limited.

Until recently, the topology optimization of fluidic flows has focused primarily on steady flows. In contrast, unsteady flows are widespread in reality, and the optimization of unsteady flows using the shape optimization method has been researched [43–45]. As such, it is desirable to extend topology optimization to the area of unsteady Navier–Stokes flows. In this paper, the density type topology optimization method is extended to unsteady incompressible Navier–Stokes flows at low and moderate Reynolds numbers based on work by Borrvall and Petersson [12] and Gersborg-Hansen et al. [22]. The optimization problem with a general objective is analyzed by the continuous adjoint method. Recently, Kreissl et al. implemented the topology optimization of unsteady flows [46]. In Kreissl's work, the optimization problem is analyzed using the stabilized SUPG and PSPG finite element formulation and the corresponding discretized adjoint equations. In this paper, the topology optimization problem with more general objectives is analyzed using the continuous adjoint method. Based on the Navier–Stokes equations and the derived continuous adjoint equations, the optimization can be implemented by choosing any stable spatial discretization method and adaptive temporal discretization method, such as the finite difference method, the finite element method or the finite volume method. In this paper, the numerical discretization of the Navier–stokes equations and the adjoint equations is implemented using the standard finite element method with Taylor-Hood element.

This paper is organized as follows: the variational problem of topology optimization for unsteady incompressible Navier–Stokes flows is stated in Section 2; the continuous adjoint equations and adjoint sensitivity for the optimization objective are derived in Section 3; the numerical implementation of the optimization method using the standard finite element method is discussed in Section 4; and several numerical examples are presented in Section 5.

2. Topology optimization problem of unsteady incompressible Navier–Stokes flows

The dynamic properties of velocity and pressure for incompressible fluidic flows can be expressed using the incompressible Navier–Stokes equations as [47]

$$\begin{aligned} &\rho\tfrac{\partial \mathbf{u}}{\partial t} - \eta\nabla\cdot(\nabla\mathbf{u}+\nabla\mathbf{u}^{\mathrm{T}}) + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \quad \text{in } Q \\ &-\nabla\cdot\mathbf{u} = 0, \quad \text{in } Q \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the fluidic velocity; p is the fluidic pressure; ρ is the fluidic density; η is the fluidic viscosity; $\mathbf{f}$ is the body force loaded on the fluid; and t is the time. $Q = (0,T) \times \Omega$, where $(0,T)$ is the computational time interval and Ω is the computational domain. To solve a transient problem, an initial condition is needed

$$\mathbf{u}(0,\mathbf{x}) = \mathbf{u}_0(\mathbf{x}), \quad \text{in } \Omega \tag{2}$$

where $\mathbf{u}_0(\mathbf{x})$ satisfies the incompressible condition $\nabla\cdot\mathbf{u}_0 = 0$. The commonly used boundary conditions for incompressible Navier–Stokes equations include the Dirichlet and Neumann type boundary conditions

$$\mathbf{u} = \mathbf{u}_D(t,\mathbf{x}), \quad \text{on } \Sigma_D \tag{3}$$

$$[-p\mathbf{I} + \eta(\nabla\mathbf{u}+\nabla\mathbf{u}^{\mathrm{T}})]\mathbf{n} = \mathbf{g}(t,\mathbf{x}), \quad \text{on } \Sigma_N \tag{4}$$

where $\mathbf{u}_D$ and $\mathbf{g}$ are the specified velocity and stress distribution on the boundaries Γ_D and Γ_N; $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$; $\Sigma_D = (0,T) \times \Gamma_D$; and $\Sigma_N = (0,T) \times \Gamma_N$. Specifically, the no-slip boundary is a particular Dirichlet type boundary condition where $\mathbf{u}_D = \mathbf{0}$, and the open-boundary on the outlet can be expressed by the Neumann type boundary condition as $\mathbf{g} = \mathbf{0}$. In topology optimization of the Navier–Stokes flow, the body force can be expressed as [12,22]

$$\mathbf{f} = -\alpha\mathbf{u} \tag{5}$$

where α is the impermeability of a porous medium. Its value depends on the optimization design variable γ [12,22]

$$\alpha(\gamma) = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min})\frac{q(1-\gamma)}{q+\gamma} \tag{6}$$

where $\alpha_{\min}$ and $\alpha_{\max}$ are the minimal and maximal values of α respectively, and q is a real and positive parameter used to adjust the convexity of the interpolation function in Eq. (6). The value of γ can vary between zero and one, where $\gamma = 0$ corresponds to an artificial solid domain and $\gamma = 1$ to a fluidic domain, respectively. Usually, $\alpha_{\min}$ is chosen as 0, and $\alpha_{\max}$ is chosen as a finite but high number to ensure the numerical stability of the optimization and to approximate a solid with negligible permeability [22,23]. When solving transient incompressible Navier–Stokes equations, the design variable γ is time independent because the layout of fluidic domain is kept unchanged.

Based on the above description, the topology optimization problem for a unsteady incompressible Navier–Stokes flow can be formulated as