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Gaussian process surrogates for failure detection: A Bayesian experimental design approach

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ABSTRACT

An important task of uncertainty quantification is to identify the probability of undesired events, in particular, system failures, caused by various sources of uncertainties. In this work we consider the construction of Gaussian process surrogates for failure detection and failure probability estimation. In particular, we consider the situation that the underlying computer models are extremely expensive, and in this setting, determining the sampling points in the state space is of essential importance. We formulate the problem as an optimal experimental design for Bayesian inferences of the limit state (i.e., the failure boundary) and propose an efficient numerical scheme to solve the resulting optimization problem. In particular, the proposed limit-state inference method is capable of determining multiple sampling points at a time, and thus it is well suited for problems where multiple computer simulations can be performed in parallel. The accuracy and performance of the proposed method is demonstrated by both academic and practical examples.

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1. Introduction

Real-life engineering systems are unavoidably subject to various uncertainties such as material properties, geometric parameters, boundary conditions and applied loadings. These uncertainties may cause undesired events, in particular, system failures or malfunctions, to occur. Accurate identification of failure modes and evaluation of failure probability of a given system is an essential task in many fields of engineering such as risk management, structural design, reliability-based optimization, etc.

In the many fields of applications such as structural engineering, system failures are often described with the limit state concepts. Simply put, the state space of the random parameters in a system is divided into two domains (while each domain may contain multiple sub-regions): the safe domain, the states in which may not cause system failures, and the failure domain the states in which may cause system failures. And the limit states represent the boundary between the two domains. The mathematical formulation of the limit states will be provided in Section 2.

Conventionally the failure probability is often computed by constructing linear or quadratic expansions of the system model around the so-called most probable point, known as the first/second-order reliability method (FORM/SORM); see e.g.,

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[33] and the references therein. It is well known that FORM/SORM may fail for systems with nonlinearity or multiple failure modes. The Monte Carlo (MC) simulation, which estimates the failure probability by repeatedly simulating the underlying system, is another popular method for solving such problems. The MC method makes no approximation to the underlying computer models and thus can be applied to any systems. On the other hand, the MC method is notorious for its slow convergence, and thus can become prohibitively expensive when the underlying computer model is computationally intensive and/or the system failures are rare and each sample requires to a full-scale numerical simulation of the system. To reduce the computational effort, one can construct a computationally inexpensive approximation to the true model, and then replace the true model with the approximate one in the MC simulations. Such approximate models are also known as response surfaces, surrogates, metamodels, and emulators, etc. These methods are referred to as the response surface (RS) methods [14,30,15] in this work. The emulator based response surface can often provide a reliable estimate of the failure probability, at a much lower computational cost than direct MC simulations.

In this work we focus on a specific kind of RS, the Gaussian processes (GP) surrogates [37], also known as kriging in many fields of applications. The GP surrogates have been applied to uncertainty quantification problems first in [25, 27] and more recently in [7,8]. A number of GP-based methods have been also been successfully implemented for failure probability estimation [21,3]. In this work we consider the situation where the underlying computer models are extremely expensive and one can only afford a very limited number of simulations. In this setting, choosing the sampling points (i.e. the parameter values with which the simulation is performed) in the state space is of essential importance. Determining the sampling points for GP can be cast as to optimally design computer experiments and considerable efforts [32] have been devoted to it. Those methods aim to construct a surrogate that can accurately approximate the target function in the whole parameter space. As will be explained later, in the failure probability estimation or failure detection problems, only the sign of the target function is used. Thus by requiring surrogates to be globally accurate, the methods may allocate considerable computational efforts to the regions not of interest, and use much more model simulations than necessary.

Several methods have developed to determine sampling points for the failure probability estimation purposes. Most of these methods consist of sequentially finding the "best point" as a result of a heuristic balance between predicted closeness to the limit state, and high prediction uncertainty, e.g. [12,6]. Such methods are shown to be effective in many applications, while a major limitation is their point-wise sequential nature, which makes it unsuitable for problems in which multiple computer simulations can be performed parallelly. An exception is the stepwise uncertainty reduction (SUR) method developed in [4,9], in which the authors proposed an optimal experimental design framework which determines multiple sampling points by minimizing the average variance of the failure probability. It should be noted that the design criteria in the SUR method is particularly developed for the goal of estimating the failure probability only. In practice, one is often not only interested in estimating the failure probability, but also identifying the events that can cause failures; the latter demands a design criteria for the goal of detecting the limit state, i.e., the boundaries of the failure domain. In this work, we recast the surrogate construction as a Bayesian inference to identify the limit state, and based on that, we formulate an *information-theoretic* optimal experimental design, which uses the relative entropy from the prior to the posterior as the design criteria, to determine the sampling points. We also present an efficient numerical scheme for solving the resulting optimal design problem, modified from the simulation-based method developed in [17]. We compare the performance of the proposed limit-state inference (LSI) method with that of the SUR by numerical examples.

We note that another line of research in failure probability estimation is to develop more efficient sampling schemes, such as the subset simulations [2], importance sampling [13], the cross-entropy method [31,36], etc. The LSI method can be easily integrated into the aforementioned sampling schemes, resulting in more efficient estimation schemes. Examples of combining surrogates and efficient sampling schemes include [20,21,11], just to name a few.

The rest of this paper is organized as following. We first review the preliminaries of our work in Section 2, including the mathematical formulation of failure probability computation and the GP surrogates. Our Bayesian experimental design framework and its numerical implementations are presented in Section 3. Numerical examples are presented in Section 4 to demonstrate the effectiveness of the proposed method, and finally Section 5 offers some closing remarks.

2. Problem formulation

2.1. Failure detection and failure probability estimation

Here we describe the failure probability estimation problem in a general setting. We consider a probabilistic model where $\mathbf{x}$ is a d-dimensional random variable that represents the uncertainty in model and let $\Omega \subseteq R^d$ be the state space of $\mathbf{x}$. The system failure is often defined using a real-valued function $g(\cdot) : \Omega \to R$, which is known as the *limit state function* or the *performance function*. Specifically, the event of failure is defined as $g(\mathbf{x}) < 0$ and as a result the failure probability is

$$P = \mathbb{P}(g(\mathbf{x}) < 0) = \int\limits_{\{\mathbf{x}\in\Omega | g(\mathbf{x})<0\}} p(\mathbf{x})d\mathbf{x} = \int\limits_{\mathbf{x}\in\Omega} I_g(\mathbf{x})p_X(\mathbf{x})d\mathbf{x},$$

where $I_g(\mathbf{x})$ is an indicator function:

$$I_g(\mathbf{x}) = \begin{cases} 1 & \text{if } g(\mathbf{x}) < 0, \\ 0 & \text{if } g(\mathbf{x}) \geq 0; \end{cases}$$

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