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# Full-wave algorithm to model effects of bedding slopes on the response of subsurface electromagnetic geophysical sensors near unconformities

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We propose a full-wave pseudo-analytical numerical electromagnetic (EM) algorithm to model subsurface induction sensors, traversing planar-layered geological formations of arbitrary EM material anisotropy and loss, which are used, for example, in the exploration of hydrocarbon reserves. Unlike past pseudo-analytical planar-layered modeling algorithms that impose parallelism between the formation's bed junctions, our method involves judicious employment of Transformation Optics techniques to address challenges related to modeling *relative* slope (i.e., tilting) between said junctions (including arbitrary azimuth orientation of each junction). The algorithm exhibits this flexibility, both with respect to loss and anisotropy in the formation layers as well as junction tilting, via employing special planar slabs that coat each "flattened" (i.e., originally tilted) planar interface, locally redirecting the incident wave within the coating slabs to cause wave fronts to interact with the flattened interfaces *as if* they were still tilted with a specific, user-defined orientation. Moreover, since the coating layers are homogeneous rather than exhibiting continuous material variation, a minimal number of these layers must be inserted and hence reduces added simulation time and computational expense. As said coating layers are not reflectionless however, they do induce artificial field scattering that corrupts legitimate field signatures due to the (effective) interface tilting. Numerical results, for two half-spaces separated by a tilted interface, quantify error trends versus effective interface tilting, material properties, transmitter/receiver spacing, sensor position, coating slab thickness, and transmitter and receiver orientation, helping understand the spurious scattering's effect on reliable (effective) tilting this algorithm can model. Under the effective tilting constraints suggested by the results of said error study, we finally exhibit responses of sensors traversing three-layered media, where we vary the anisotropy, loss, and *relative* tilting of the formations and explore the sensitivity of the sensor's complex-valued measurements to both the magnitude of effective relative interface tilting (polar rotation) as well as azimuthal orientation of the effectively tilted interfaces.



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## 1. Introduction

Long-standing and sustained interest has been directed towards the numerical evaluation of electromagnetic (EM) fields produced by sensors embedded in complex, layered-medium environments [1]. In particular, within the context of geophysical exploration (of hydrocarbon reserves, for example), there exists great interest to computationally model the response of induction tools that can remotely sense the electrical and structural properties of complex geological formations (and consequently, their hydrocarbon productivity) [2,3]. Indeed, high-fidelity, rapid, and geometry-robust computational forward-modeling aids fundamental understanding of how factors such as the formation's global inhomogeneity structure, conductive anisotropy in formation bed layers, induction tool geometry, exploration borehole geometry, and drilling fluid type (among other factors) affect the sensor's responses. This knowledge informs both effective and robust geophysical parameter retrieval algorithms (inverse problem), as well as sound data interpretation techniques [2,4]. Developing forward-modeling algorithms which not only deliver rapid, accuracy-controllable results, but also simulate the effects of a greater number of dominant, geophysical features without markedly increased computational burden, represents a high priority in subsurface geophysical exploration [5,6].

In the interest of obtaining a good trade-off between the forward modeler's solution speed while still satisfactorily modeling the EM behavior of the environment's dominant geophysical features, a layered-medium approximation of the geophysical formation often proves very useful. Indeed cylindrical layering, planar layering, and a combination of the two (for example, to model the cylindrical exploratory borehole and invasion zone embedded within a stack of planar formation beds) are arguably three of the most widely used layering approximations in subsurface geophysics [2–5,7–25], for both onshore and offshore geophysical exploration modeling [26–36]. The prevalence of layered-medium approximations stems in large part, at least from a computational modeling standpoint, due to the typical availability of closed-form eigenfunction expansions to compute the EM field [37, Ch. 2–3]. These full-wave techniques are quite attractive since they can *robustly* deliver rapid solutions with high, user-controlled accuracy under widely varying conditions with respect to anisotropy and loss in the formation's layers, orientation and position of the electric or (equivalent) magnetic current-based sensors (viz., electric loop antennas), and source frequency [22,36]. The robustness to physical parameters is highly desirable in geophysics applications since geological structures are known to exhibit a wide range of inhomogeneity profiles with respect to conductivity, anisotropy, and geometrical layering [2,6,12]. For example, with respect to formation conductivity properties, diverse geological structures can embody macro-scale conductive anisotropy in the induction frequency regime, such as (possibly deviated) sand-shale micro-laminate deposits, clean-sand micro-laminate deposits, and either natural or drilling-induced fractures. The electrical conduction current transport characteristics of such structures indeed are often mathematically described by a uniaxial or biaxial conductivity tensor exhibiting directional electrical conductivities whose value range can span in excess of two orders of magnitude [2,7,14].

When employing planar and cylindrical layer approximations, one almost always assumes that the interfaces are parallel, i.e., exhibit common central axes (say, along $z$) in the case of cylindrical layers [24,25], or interfaces that are all parallel to a common plane in the case of planar layers [3,12]. However, it may be more appropriate in many cases to admit layered media with material property variation along the direction(s) conventionally modeled as homogeneous. For example, in cylindrically-layered medium problems involving deviated drilling, gravitational effects may induce a downward diffusion of the drilling fluid that leads to a cylindrical invasion zone angled relative to the cylindrical exploratory borehole [6]. Similarly, formations that locally (i.e., in the proximity of the EM sensor) appear as a "stack" of beds with *tilted* (sloped) planar interfaces can appear (for example) due to temporal discontinuities in the formation's geological record. These temporal discontinuities in turn can manifest as commensurately abrupt spatial discontinuities, known as unconformities (especially, angular unconformities) [38–40]; see Fig. 1a below for a schematic illustration. Indeed, the effects of unconformities and other complex formation properties (such as fractures) have garnered increasing attention over the past ten years [41–43], particularly in light of the relatively recent availability of induction sensor systems offering a rich diversity of *simultaneous* measurement information with respect to radiation frequency, transmitter and receiver orientation ("directional" diversity), and transmitter/receiver separation [43–47].

A natural question arises as to which numerical technique is best suited to modeling these more complex geometries involving tilted layers. In principle, one could resort to brute-force techniques such as finite difference and finite element methods [3,16,46,48,49]. The potential for low-frequency instability (e.g., when modeling geophysical sensors operating down to the magnetotelluric frequency range [fraction of a Hertz]), high computational cost (unacceptable, especially when many repeated forward solutions are required to solve the inverse problem), and accuracy limitations due to mesh truncation issues (say, via perfectly matched layers or other approaches [50,51]) associated with the lack of transverse symmetry in the tilted-layer domain [52], render these numerical methods less suitable for developing fast forward-modeler engines for tilted-layer problems. Another potential approach involves asymptotic solutions which traces the progress of incident rays and their specular reflections within subsurface formations [53]. However, this approach is limited to sufficiently high-frequencies and hence unsuitable for modeling low-frequency sensors operating in zones where highly resistive and highly conductive (not to mention anisotropic) layers may coexist [54]. Yet a third possible approach, called the "Tilt Operator" method, which assumes lossless media and negligible EM near-fields to avoid spurious exponential field growths (arising from violation of "primitive" causality [i.e., cause preceding effect], which is inherent in this method), is another possibility [55,56]. Akin to the other mentioned high-frequency approach however [53], the Tilt Operator method is not appropriate for our more general class of problems with respect to sensor and geological formation characteristics.