Journal of Computational Physics 313 (2016) 377–399

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A loosely-coupled scheme for the interaction between a fluid, elastic structure and poroelastic material

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ARTICLE INFO

Article history:
Received 2 March 2015
Received in revised form 19 February 2016
Accepted 20 February 2016
Available online 26 February 2016

Keywords:
Fluid–structure interaction
Poroelasticity
Fluid–porous media interaction
Loosely-coupled scheme
Blood clots

ABSTRACT

We model the interaction between an incompressible, viscous fluid, thin elastic structure and a poroelastic material. The poroelastic material is modeled using the Biot's equations of dynamic poroelasticity. The fluid, elastic structure and the poroelastic material are fully coupled, giving rise to a nonlinear, moving boundary problem with novel energy estimates. We present a modular, loosely coupled scheme where the original problem is split into the fluid sub-problem, elastic structure sub-problem and poroelasticity sub-problem. An energy estimate associated with the stability of the scheme is derived in the case where one of the coupling parameters, β, is equal to zero. We present numerical tests where we investigate the effects of the material properties of the poroelastic medium on the fluid flow. Our findings indicate that the flow patterns highly depend on the storativity of the poroelastic material and cannot be captured by considering fluid–structure interaction only.

1. Introduction

Poroelastic materials, due to their dissipative properties, are extensively used in the automotive industry, aeronautics, and building acoustics for the purpose of noise control. They are also ubiquitous in nature. Examples include bones, soil, blood clots and biological tissues. In practical applications poroelastic materials are involved in multilayered structures comprising elastic media. These materials are exposed to a wide spectrum of dynamic loads. In the aerospace industry, dam–reservoir systems, off-shore structures and in many biological systems, the dynamic loading comes from the surrounding fluid such as air, water or blood flow. Understanding the interaction between a fluid, elastic structure and a poroelastic material is important for the understanding of the normal function and prevention of damage and catastrophic events in many applications. One of the applications of such systems in hemodynamics is the interaction of a blood clot with blood flow dynamics and vessel wall mechanics. Understanding their interaction plays an important role in making decisions about medical treatment, since it helps identify flow conditions which may lead to medical complications. In this work, we focus on this application and show that accurate flow patterns cannot be captured without including the dynamics of the blood clot in fluid–structure interaction models.

Blood clots contain different types of blood [1], cells [32] and extracellular matrix constituents. Near the vessel wall, they show a considerable amount of collagen [44], forming a core with a low permeability. The layers around the core contain mainly fibers, overlaid by a micro-structure of pores and cavities filled by a fluid [1,36] whose dimensions vary strongly. Blood clots have been shown to be permeable to fluids, which potentially allows the passage of macromolecules to the vessel wall [1]. Hence, the blood clot dynamics on a macro scale have been previously described using porous media

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http://dx.doi.org/10.1016/j.jcp.2016.02.051

models [26,65], and poroelasticity models [57]. However, none of these approaches considered the full interaction between blood, blood clot and the vessel wall. The work presented here is the first step in that direction.

In the existing literature, problems involving the interaction between a fluid, elastic structure and poroelastic material are predominantly studied in terms of the interaction of poroelastic media with an elastic structure and an air-filled acoustic region [3,59,4,24,67,48,46,43]. In contrast to the existing work related to acoustic wave propagation, we are interested in the interaction between an incompressible, viscous fluid, an elastic structure and a poroelastic material with applications to hemodynamics. The novel, two-way coupling gives rise to a non-linear, moving boundary problem with energy estimates not considered before. Since the coupled problem features interaction between different physical phenomena, defined on separate domains, we develop and analyze a loosely-coupled numerical algorithm that captures this interaction. We split the fully coupled problem into a fluid sub-problem, elastic structure sub-problem and poroelasticity sub-problem. At every time step, each of these sub-problems is solved only once, without additional iterations between the sub-problems, leading to a loosely-coupled scheme. To derive the numerical scheme, we extend and combine splitting strategies for fluid–structure interaction (FSI) problems and fluid–poroelastic material interaction (FPI) problems.

The development of partitioned numerical methods for FSI problems has been extensively studied in the literature [11,6,52,28,27,55,39,8,58,47]. However, one of the main concerns of partitioned schemes is stability. It has been shown in [20] that "classical" partitioned strategies suffer from numerical instabilities known as the added mass effect when applied to problems in hemodynamics. To resolve this issue, several alternative methods have been proposed. Methods proposed in [31,55] use a membrane model for the structure that is then embedded into the fluid problem where it appears as a generalized Robin boundary condition. We further mention the numerical schemes proposed in [6,5] where the fluid and structure were split in the classical way, but the coupling conditions were combined in a novel way that improved the convergence rate. A different approach based on Nitsche's penalty method [39] was used in [14,15].

Recently, a loosely coupled scheme, called the "kinematically coupled β-scheme," was introduced in [11]. This scheme successfully deals with stability problems associated with the added mass effect in a novel way. The scheme originates from the "classical" kinematically coupled scheme introduced in [38], where a parameter β was introduced in [11] to increase the accuracy of the scheme. The kinematically coupled scheme introduced in [38] corresponds to the case when $\beta = 0$. Taking $\beta > 0$, and in particular $\beta = 1$, is shown to increase the accuracy without affecting the stability [11]. Stability and accuracy results on a nonlinear benchmark FSI problem in [11] show that the accuracy of the kinematically coupled β–scheme with $\beta = 1$ is comparable to the accuracy of a monolithic scheme by Badia, Quaini, and Quarteroni [7]. Stability was further studied analytically in [16], where unconditional stability for any $\beta \in [0, 1]$ was shown. The stability is achieved by combining the structure inertia with the fluid sub-problem as a Robin boundary condition in a novel way, which mimics the energy balance of the continuous problem.

A class of incremental displacement-correction schemes for the explicit coupling of FSI problems has been introduced in [29,28]. The non-incremental variant of the algorithm presented there corresponds to the kinematically coupled scheme, while the incremental scheme consists of treating the structure displacement explicitly in the fluid sub-problem and then correcting it in the structure sub-problem. The authors in [28] presented stability and convergence analysis for FSI problems with thin structures, for the whole class of incremental displacement-correction schemes, showing that the incremental scheme features optimal convergence, while sub-optimal convergence is expected for the original non-incremental variant. In [29] the authors proposed a generalization of the explicit coupling schemes reported in [28,38] to the case of FSI with thick structures.

The kinematically coupled β scheme was also used in a modified version by Lukacova et al. in [47,42] to study FSI involving non-Newtonian fluids, and by Bukač et al. in [12] to model the interaction between a fluid and a composite structure. These recent results indicate that the kinematically-coupled β scheme and its modifications provide an appealing way to solve FSI problems using a partitioned approach and therefore we employ it in this work.

In addition to the interaction between fluid and an elastic structure discussed above, the model we study also features the interaction between the fluid and a poroelastic material. A poroelastic material consists of an elastic skeleton and connecting pores filled with fluid. A common way to describe such system is using a Biot model. Due to the various geomechanical applications, where the structure dynamics is negligible, the quasi-static Biot model is well studied in the literature [64,61,62,56,49]. However, since we are interested in hemodynamics applications in which the structure dynamics are significant, we consider the dynamic Biot model [21,66].

The coupling between a fluid and a poroelastic medium has been previously studied in [34,8,45,60,53,63,37,35,13]. The formulation, boundary conditions and well-posedness were studied in [60]. Numerical studies include the work in [8], where the FPI problem is solved using both a monolithic and a partitioned approach. The partitioned approach is based on the domain decomposition procedure. However, sub-iterations are needed between the two problems due to the instabilities associated with the added mass effect. Studies in [37,35] considered coupling between a poroelastic material and a reduced fluid model described by the lubrication equation in order to study the flow in fractures. Particular applications of the FPI model were studied in [45], where such a system was used to describe the blood flow and low-density lipoprotein transport in a porohyperelastic arterial wall, and in [63] to model cerebrospinal fluid hydrodynamics.

In this work, in order to split the fluid problem from the dynamic poroelasticity problem, we extend and adapt the splitting strategy applied in [18] to the Stokes/Darcy interaction problem. Furthermore, similar to [8], we introduce combination parameters, which are then combined with the splitting used in [18], and applied to the interaction between the fluid and