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A multimaterial extension to subzonal reconstruction

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ABSTRACT

We present a new algorithm for reconstructing material-dependent subzonal information based on staggered primary/dual-mesh remapping of material-dependent conserved quantities. The algorithm is appropriate in the context of geometric, intersection-based overlay remapping methods, with specific application to staggered, total energy conserving, multi-material Lagrangian hydrodynamics schemes that discretize material masses on subzonal mesh elements. Our new approach avoids direct remapping of material-dependent subzonal variables; instead, the spatial profile of each variable is reconstructed using a combination of material-dependent zone (primary mesh) information, material-independent node (dual mesh) information, and discrete interface-reconstructed material concentration information. Conservation and convergence properties of the new algorithm are established through several challenging multi-material remapping and hydrodynamics tests.

1. Introduction

In [18], we outlined an algorithm for conservatively reconstructing subzonal information using staggered data at zones (primary mesh) and nodes (dual mesh). Because the number of subzonal elements in the mesh exceeds the total number of zone and node elements, reconstruction takes the form of a least-squares optimization procedure. Conservation is achieved by enforcing linear equality constraints at each zone and node.

This work is motivated by staggered, compatible [5] Arbitrary Lagrangian–Eulerian (ALE) methods that employ an overlay-based (also known as *intersection*, *interpolation*, or *geometric*) remap. In the staggered, compatible framework, mass is discretized onto subzonal elements to ensure consistent representations of mass at both zones and nodes. Conservation of mass, linear momentum, and energy is a natural consequence of this discretization.

In [13], the authors propose a separate method for remapping subzonal data. State variables are collocated to subzonal elements and remapped from subzones to subzones. We initially proposed the reconstruction algorithm in [18] to alleviate the expense of computing subzone–subzone intersections for each remap. The computational cost of an overlay remap scales with the number of element intersections. Timing data and scaling arguments in [18] indicate that staggered overlay remapping (i.e. computing zone–zone and node–node intersections) followed by subzonal reconstruction outperforms direct subzonal remapping in terms of computational efficiency for 2D polygonal meshes.

A second motivation for pursuing subzonal reconstruction over direct subzonal remapping is the problem of remapping multimaterial data. The direct subzonal remap described in [13] and applied in [12] is only applicable for single-material problems. Extending the methodology to multi-material data in an efficient manner is not straightforward; at the very least, a subzonal remap requires a multi-material model that is discretized onto subzonal elements. In the context of multi-

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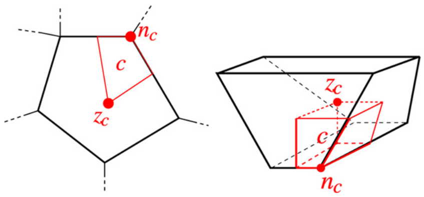

Fig. 1. Corner subvolumes (c, in red) of a 2D (left) and 3D (right) zone. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

material ALE calculations, this implies storing discrete volume and/or mass fraction data at subzones as well as performing interface reconstruction at a sub-grid scale. In our experience, most multi-material hydro codes discretize material concentrations on zone elements. Adapting such codes to handle the higher memory and complexity requirements of a fully subzonal multi-material discretization is not optimal—rather we prefer to adapt our remap algorithm to handle zone centered material fractions.

In this paper, we describe a multimaterial extension to the subzonal reconstruction algorithm. The extension is conservative and requires minimal modification of the host code to operate. We consider a volume-of-fluid (VOF) multimaterial model with volume fraction data discretized onto zonal elements [1]. In principle, the algorithm may be adapted to operate within the moment-of-fluid (MOF) multimaterial framework as well with little modification.

Note that the problem of remapping multimaterial subzonal masses in a conservative, monotonic, and efficient manner is not confined to overlay-based remaps. Similar problems arise in advective/flux-based remaps as well. The availability of numerical mass fluxes simplifies the problem somewhat. Interested readers may consider [15] for advective remapping strategies for staggered, compatible discretizations.

Hybrid methods have also been developed recently to bridge the gap between overlay and advective remaps. Numerical mass fluxes may be obtained from intersection data and limited in intelligent ways to promote conservation, monotonicity, and accuracy. In the context of this work, we point the reader toward [9,10].

The paper is organized as follows: section 2 describes the compatible subzonal discretization of hydrodynamic variables; section 3 defines the overlay procedure for remapping single- and multi-material data; section 4 details the three-stage multimaterial subzonal reconstruction algorithm; and section 5 demonstrates the new algorithm's performance a series of pure remapping and ALE hydrodynamics test problems.

2. Background

2.1. Subzonal discretization

Our main application in this paper is the use of a geometric overlay-based remap with the spatially-staggered, Lagrangian-frame algorithm for hydrodynamics outlined in [5]. The Lagrangian hydrodynamic discretization of [5] derives a number of important properties—for instance, conservation, generality to unstructured grids, and improved accuracy—from the discretization of state variables on subzonal mesh elements.

Each zone is subdivided into subzonal elements, often referred to as *corners*, indexed here by c. Each corner is a polyhedron in 3D or a quadrilateral in 2D and corresponds to a unique neighboring zone–node pair (z_c, n_c) on an arbitrary, unstructured mesh. The vertices of corner c consist of the centroid of zone z_c, the position of node n_c, and the centers of each of the faces and edges of z_c that touch n_c (see Fig. 1).

Element volumes for zones V_z and nodes V_n are defined by summing neighboring corner volumes V_c:

$$V_z = \sum_{c(z)} V_c \quad \text{and} \quad V_n = \sum_{c(n)} V_c. \tag{1}$$

Here $c(z)$ denotes the set of corners inside zone z, and $c(n)$ denotes the set of corners touching node n. The union of each corner element in $c(n)$ defines the mesh element associated with node n (see Fig. 2). A corner can thus be thought of as the intersection of zone element z_c on the primary mesh and node element n_c on the dual mesh.

2.2. Subzonal densities and compatible hydrodynamics

The staggered, compatible discretization for conservative hydrodynamics places kinematic variables of velocity and kinetic energy at nodes and thermodynamic variables of internal energy at zone centers. In order to facilitate Lagrangian definitions of the mass on both zones and nodes, the fluid mass is stored on the subzonal corners. Discrete zonal and nodal masses are then naturally defined as coarsened representations of these subzonal corner masses on their respective element volumes:

$$m_z = \sum_{c(z)} m_c \quad \text{and} \quad m_n = \sum_{c(n)} m_c, \tag{2}$$