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A controllable canonical form implementation of time domain impedance boundary conditions for broadband aeroacoustic computation

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ABSTRACT

A new method, which can be effectively and efficiently applied in the simulations of broadband noise problems, is proposed for time domain impedance boundary condition implementations by using the so-called controllable canonical form that is well known in linear system. Usually, the impedance boundary condition can be defined in frequency domain as a rational polynomial function with poles in the negative half of the complex plane to guarantee stability; otherwise, causality might be violated in the corresponding time domain implementation. To address this issue, various methodologies have been proposed previously that usually lead to complicated polynomials, whose numerical implementations are often indirect and intricate. The proposed method with a controllable canonical form, on the other hand, directly transforms the frequency domain transfer function (a quotient of rational polynomials) to an equivalent state space model, which consists of a series of first-order ordinary differential equations that can be numerically implemented in a straightforward way. The proposed method is demonstrated by using two benchmark problems: a two-dimensional Gaussian pulse propagating in a uniform flow with a lined wall and the test cases from the NASA Langley grazing incidence tube experiments. Good agreements demonstrate the potential of the proposed computational method.

1. Introduction

To meet the increasingly stringent regulation [15] and certification requirements, various means of acoustics attenuation are being explored. Among them acoustic liners are widely used in aero-engines to reduce noise emissions [24,35]. The most important property of a liner is the associated acoustic impedance, $Z(\omega)$, which is usually defined in frequency domain as the ratio of the local sound pressure to the normal particle velocity on the lining surface. Optimization of the liner impedance is usually so time-consuming that calls for continuous developments of numerical methods [3], especially in the topics of efficient solvers, effective optimization methods, and robust and accurate impedance boundary conditions (see Fig. 1). This paper is mainly focused on the numerical implementation of the impedance boundary conditions.

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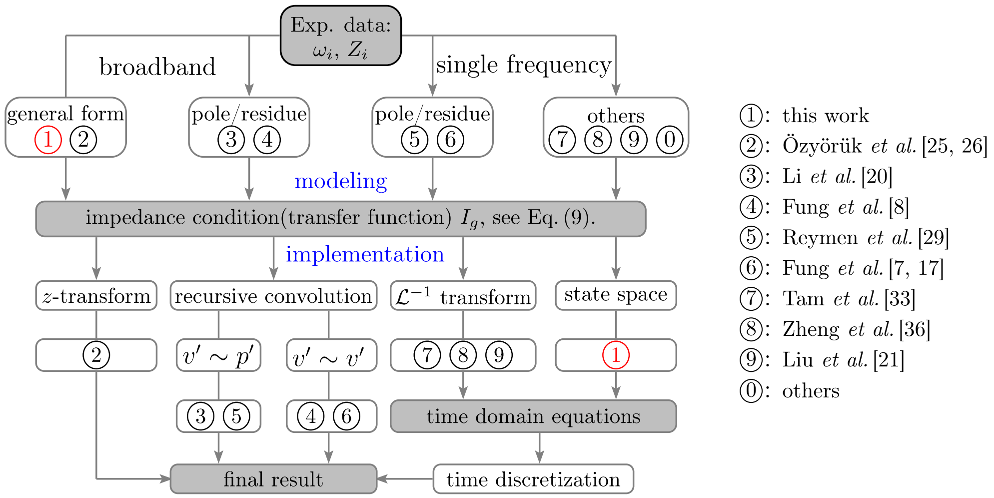

Fig. 1. Classification of time domain impedance boundary conditions.

A time domain solver is usually preferred for its capability in efficiently solving broadband, transient, possible nonlinear and multi-dimensional problems [11]. An important ingredient in a time domain solver is an appropriate time domain impedance boundary condition, which should implement the corresponding representations in frequency domain that can be achieved by curve fitting of the available experimental data. However, a direct implementation often calls for inconvenient convolution operation that might even violate causality [31]. This key issue has attracted a large amount of attentions during the last three decades.

Tam and Auriault [33] proposed a causal time domain impedance boundary condition for a stationary background flow by employing a mathematical mapping. Li et al. [19] and Zheng and Zhuang [36] extended this work to subsonic flow cases. Reymen et al. [29] suggested using recursive convolution, which was to some degree inspired by the linear system theory. Based on a similar recursive method, Li et al. [20] proposed an improved multipole treatment that decomposed the rational polynomial of impedance into a series of residues and poles to approximate the frequency response. Following a different routine, Fung and Ju [7,8,17] described an impedance model by using the equivalent concept of reflection coefficients, which implicitly ensured the computational stability since the reflection coefficients are always less than 1.0. Most of these previous works actually focused on generating a causal transfer function (to represent acoustic impedance). Nevertheless, the follow-up implementations in the time domain usually request complicated and specialized treatments.

Özyörük and Long [25] and Özyörük et al. [26] proposed a straightforward numerical method from a transfer function to the discrete time domain implementation for broadband computations by using z-transform, which is usually used in digital control and digital signal processing [32]. More specifically, given a causal transfer function, one can obtain the equivalent representation in the discrete time domain by performing bilinear transform, which will preserve the stability properties of the original transfer function [32]. The resultant implementation is extremely straightforward by replacing z^{-1} by a single time step delay. However, it is a common practice in computational aeroacoustics to use multi-stage and multi-step time integration schemes, in which cases the z-transform based design will become complicated. Moreover, as will be shown below, spatial derivatives appearing in the impedance boundary conditions will further complicate the numerical implementation.

The above works adopted the so-called Ingard–Myers model to take account of uniform flow effects. The viscous boundary layer of a liner was usually assumed vanishingly thin, so that a ‘fictious’vortex sheet was assumed to separate the nonslip lining surface and the outer mean flow region. However, it is well known that this model is ill-posed and the corresponding time domain implementation might yield nonphysical spatial amplification [5]. Brambley and Gabard [5] proposed to solve this issue of ill-posedness by taking the boundary layer thickness into consideration. By following a different strategy, Liu et al. [21] proposed to remove nonphysical instabilities in time domain aeroacoustics simulations by including a compensator, which is designed based on classical control principles. Both strategies yield complicated transfer functions in frequency domain and the corresponding time domain implementations require tricky treatments.

Fig. 1 summarizes all the aforementioned previous works. It can be seen that different methods are categorized according to their characteristics in the modeling and implementation strategies. It becomes clear that a number of methods have been already available to stabilize a transfer function of the impedance model before the corresponding time domain implementation. The latter procedure, however, is still complicated, usually requiring some special treatments. This work is primarily focused on solving this issue. A straightforward time domain implementation for a generic and causal represented impedance boundary condition in the form of rational function is proposed here by using the state space canonical form. The technique used in this paper is mainly inspired by the linear control theory [32] while new problems appeared in this work will be addressed below to take account of the flow effects and spatial derivatives in the sound wave propagation model.