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A frozen Gaussian approximation-based multi-level particle swarm optimization for seismic inversion

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ABSTRACT

In this paper, we propose a frozen Gaussian approximation (FGA)-based multi-level particle swarm optimization (MLPSO) method for seismic inversion of high-frequency wave data. The method addresses two challenges in it: First, the optimization problem is highly non-convex, which makes hard for gradient-based methods to reach global minima. This is tackled by MLPSO which can escape from undesired local minima. Second, the character of high-frequency of seismic waves requires a large number of grid points in direct computational methods, and thus renders an extremely high computational demand on the simulation of each sample in MLPSO. We overcome this difficulty by three steps: First, we use FGA to compute high-frequency wave propagation based on asymptotic analysis on phase plane; Then we design a constrained full waveform inversion problem to prevent the optimization search getting into regions of velocity where FGA is not accurate; Last, we solve the constrained optimization problem by MLPSO that employs FGA solvers with different fidelity. The performance of the proposed method is demonstrated by a two-dimensional full-waveform inversion example of the smoothed Marmousi model.

1. Introduction

In seismic inversion, full-waveform inversion (FWI) [28,41] is a promising technique to reconstruct subsurface velocity profiles from seismograms. The method becomes increasingly popular thanks to its ability to produce high resolution images of the velocity profile. FWI is usually cast as minimizing the misfit between the collected data and prediction provided by the simulation of seismic wave propagation.

Despite the rather simple formulation, computation of FWI problems is rather challenging. One of the major difficulties lie in that the minimization problem is highly non-convex [37]: most gradient based optimization techniques rely on good initial guesses to reach global minima, and unfortunately such good initial guesses are often not available in practical problems. In fact, usually one first finds an approximate FWI solution with global optimization techniques and then improves it with gradient-based optimization methods. Particularly, stochastic global optimization techniques, such as simulated annealing (SA) [35,39], genetic algorithm (GA) [33,36], and more recently, particle swarm optimization (PSO) [34], have been

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applied to seismic inversion problems due to their abilities of escaping from undesired local minima. However, stochastic optimization methods usually require a large number of repeated simulations of wave propagation, and each simulation involves waves of high-frequency, whose wavelengths are extremely short compared to the domain size of interest. As a result, direct simulations of high-frequency wave propagation, e.g., finite difference/volume/element methods, can be extremely computationally expensive. Thus performing a global optimization with direct simulations of high-frequency waves is nearly prohibitive.

Alternative approaches to reduce the computational cost are to look for approximate solutions to wave equation based on semiclassical approximation, among which, the ray theory [2,31,6] provides attractive alternatives. In the ray-based approaches, one decomposes wavefields into elementary waveforms which propagate along rays, and reconstructs wavefields based on the dynamic information on rays (e.g., path trajectory, amplitude and phase). Kirchhoff migration [10,21] and Gaussian beam migration [15,16,27,11,12,30] are famous seismic migration methods of this kind. Kirchhoff migration greatly increases the computational speed with an asymptotic error proportional to the ratio of wavelength over the domain size, but it yields unbounded amplitudes at caustics. Gaussian beam migration keeps the merits of ray tracing, but also handles multipathing as well as maintain accuracy at caustics. However, Gaussian beam migration relies on the Taylor expansion around the central ray, hence the error of the approximation increases when the beams become wide; see Example 4.2 in [23] for a numerical study of this case. One needs to tune the width parameter of Gaussian beams in order to get a good resolution, especially when the wave solution spreads over time [3,15,7]. This is practically difficult due to the heterogeneity of the media and the non-linearity of the Riccati equation involved in the beam construction.

Recently, the frozen Gaussian approximation (FGA) method [42] was developed for seismic modeling in complex structures. The method was originally motivated by Herman–Kluk propagator for solving the Schrödinger equation in quantum chemistry [14,19,20], and later generalized to linear strictly hyperbolic systems [23–25]. The main idea of FGA is to use Gaussian functions with fixed widths to approximate the solution of wave equation. These Gaussian functions are also called coherent states in quantum mechanics, which was previously applied in seismic imaging [1,9] but did not have the rigorous treatment of amplitude factors given by FGA. Compared to Gaussian beam migration, FGA can provide a more accurate and robust solution, especially in the situation of wave spreading [23,24]. The main procedure of FGA is described as following. The initial data are decomposed into a sum of Gaussians with fixed (small) widths. Then one propagates each Gaussian function along geometric rays. The amplitudes of these Gaussians are given according to rigorously-derived dynamic equations so that the sum of them produces a good approximation to the solution of wave equation at the final time.

As an asymptotic solution to the wave equation, the accuracy of FGA is derived in [24]. Nevertheless, the actual asymptotic error of FGA depends on the smoothness of velocity profiles and it loses accuracy in certain cases (e.g. discontinuous media), which will lead to erratic inversion results. In practice, it is desirable to restrict the search within the region of smooth velocities where FGA provides an accurate approximation, however such a region cannot be easily identified *a priori*. In this work, we introduce a constraint in the original FWI problem to prevent the solution moving out of the FGA-valid region. This will add proper smoothing effects, and yield a smoothed velocity profile for the original FWI problem. It is also consistent with the original problem when the wave equation is solved exactly.

In principle all the aforementioned global optimization techniques such as SA and GA can be used to solve the constrained optimization problem. Here we choose the PSO method for its reported superior computational efficiency in various applications [13]. Another issue in optimization is the computational cost of FGA. As will be shown in Section 2, the accuracy of FGA depends on the initial approximation error, which in turn can only be reduced by increasing the number of Gaussians in the initial decomposition. Consequently, the cost of computing a highly accurate FGA solution can be rather high (though still much lower than direction simulations). On the other hand, at the early stage of the optimization, high-accuracy solutions may not be necessary, which motivates the idea to start with a less accurate FGA solver and gradually increase its precision as the optimization proceeds. Accordingly, we propose a multi-level particle swarm optimization (MLPSO) algorithm that employs FGA solvers with different fidelity (namely, different number of beams) to further improve the computational efficiency.

In summary, the main contribution of the work is threefold: we propose to use FGA to accelerate the computation of high-frequency FWI problems; we design a constrained optimization problem to prevent the solution getting into the region where FGA is not accurate; we develop an MLPSO algorithm to efficiently solve the resulting optimization problem.

The rest of this paper is organized as follows. In Section 2, we describe the FGA algorithm for the computation high-frequency wave propagation. We introduce the constrained FWI optimization problem in Section 3. In Section 4, we present the MLPSO algorithm to solve the constrained problem. The performance of our method is demonstrated in Section 5 by a two-dimensional example of smoothed Marmousi model. Finally, we make conclusive remarks in Section 6.

2. Frozen Gaussian approximation

We consider a high-frequency acoustic wave equation in d-dimension:

$$\frac{\partial^2 u}{\partial t^2} - \xi(\boldsymbol{x})\,\Delta u = 0, \tag{2.1}$$

with the prescribed initial conditions: