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An immersed-boundary method based on the gas kinetic BGK scheme for incompressible viscous flow

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ABSTRACT

In this paper, an immersed-boundary (IB) method based on the gas kinetic Bhatnagar–Gross–Krook (BGK) scheme is proposed to simulate the incompressible viscous flow around stationary and moving rigid bodies. In the presented IB method, a set of Lagrangian points represent the solid boundary and will exert force on the surrounding Eulerian points which are the cell centers within the finite-volume framework. This force is calculated by a special iterative procedure and has an effect on the flux at the interface of cell, which guarantees the fulfillment of the no-slip boundary condition and entirely avoids the flow penetration. The flow field is obtained from the BGK scheme with local grid refinement. Without complex mesh generation and transformation, the present technique can be conveniently applied to simulations with complex and moving solid boundaries. The second-order temporal accuracy and better than first-order spatial accuracy in L_2 norm are testified by the simulation of Stokes' first problem. Other three different test cases, including flows around a stationary circular cylinder, two circular cylinders in tandem and an oscillating circular cylinder, demonstrate the good capability of the present method.

1. Introduction

In recent years, many numerical schemes based on the gas-kinetic theory have been developed. The Bhatnagar–Gross–Krook (BGK) scheme [1,2] is one of the typical representatives. The scheme is robust, positivity-preserving and satisfies the entropy condition spontaneously. In smooth flow region it is just equal to the scheme based on Navier–Stokes (NS) function, and in the discontinuous region it can automatically introduce proper viscosity. Due to the above property, the BGK scheme is increasingly widely used.

One advantage of the BGK scheme is that its computational process is the same as traditional finite-volume method, which involves reconstruction of initial data, gas evolution and projection three stages. The flow information is stored and refreshed as macroscopic variables just like in the macroscopic method. So a series of techniques adopted by the traditional macroscopic method, such as reconstruction techniques, boundary techniques and acceleration techniques, can be easily expanded to the BGK scheme with just a little modification. In this paper, we have combined the BGK scheme with the immersed-boundary (IB) method. Utilizing this powerful boundary technique, we can handle more problems by BGK scheme.

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The IB method may be the simplest way to deal with complex as well as moving solid boundaries. In an IB method, a simple Cartesian grid is used and the solid boundary will have an effect on the adjacent grid points to fulfill the boundary condition. The grid doesn't have to fit the solid body, which makes the grid generation easier than a conventional method. This also means that the quality of the grid can be good although the solid boundary may be complex. Another benefit is that the IB method is very suitable to handle moving boundaries. Compared to the body-fitted or unstructured grid method, no grid transformation or regeneration is involved in the IB method, which will decrease the computational cost significantly. There is also another method which uses a Cartesian grid: the so-called Cartesian grid method [3–6]. In this method, the cells around the body surface are cut by the boundary and the grid region that lies in the solid body is discarded. Due to the irregular shape of the cut cells, special treatment must be done and this may degrade the computational efficiency and accuracy.

Peskin [7] first used the IB method to compute the blood flow around heart valves in 1972. Since then, a lot of modifications have been made and numerous variants were proposed. Although the IB method was first developed to tackle the elastic boundary condition, it is also widely used in the rigid-boundary case. Goldstein et al. [8] proposed the virtual boundary method in the spectral method framework to simulate the flow with rigid boundaries. In the virtual boundary method, the solid boundary is represented by a set of points coincide with grid points. The forces on the boundary points are first calculated by a feedback method based on the difference between the actual velocity and the predicted velocity of the boundary, and then distributed to the adjacent grid points by a narrow Gaussian distribution. Saiki and Biringen [9] extended the virtual boundary method to the finite difference formulation. The boundary points no longer coincide with the grid points and the use of finite differences eliminates the spurious oscillations caused by the feedback forcing term. The method of Lai and Peskin [10] can also be seen as a special version of the virtual boundary method by modifying the feedback mechanism a little [11].

One fatal defect of the above IB methods is that the forcing term is pre-determined, which will severely limit the time step and cannot exactly guarantee the no-slip condition. Shu et al. [12] discussed this issue and proposed a new approach to eliminate the flow penetration. Wu et al. [13] absorbed Shu et al.'s [12] idea and proposed their immersed boundary-lattice Boltzmann method, where the forcing term takes the form of velocity correction and is calculated by an implicit way which exactly guarantees the no-slip condition. Similar idea can be found in Su et al. [14], Taira and Colonius [15] and Le et al. [16], where the force is determined implicitly by the no-slip condition and the Dirac delta function is used to link the Eulerian and Lagrangian variables.

In addition to the above IB methods, which are generally categorized to continuous forcing approach, there is the other group called direct forcing approach [11]. Mohd-Yusof [17] proposed his direct forcing method in a spectral framework. In Mohd-Yusof's method, a mirrored velocity field is formed in the solid region, which means that the velocity boundary condition is directly imposed on the Eulerian points adjacent to the surface inside the body. Then the force is obtained through the momentum equation. Fadlun et al. [18] modified this method and applied it to a three-dimensional finite-difference framework. In the method of Fadlun et al. the forcing points are external to the body and a different reconstruction strategy is adopted. Other modifications for the direct forcing approach can be found in [19–21].

In our work, we absorb the ideas of Wu et al. [13] and some other previous IB methods [14–16] and apply them to the BGK scheme in the finite-volume framework. In our method, the solid boundary is represented by a set of Lagrangian points while the fluid domain is represented by the Eulerian points, i.e. the cell centers. The discrete delta function is used to link the Eulerian and the Lagrangian variables. The force on the Lagrangian point is calculated by an iterative procedure and distributed to the surrounding Eulerian points, which will make the interpolated velocity of the Lagrangian point finally satisfies the no-slip condition. The force on the Eulerian point will influence the flux at the cell's interface through the BGK flux solver and the flow field is obtained from the BGK scheme with external force term. A local Quadtree refinement technique is employed to decrease the computational cost. The temporal and the spatial accuracy of the present method are analyzed by the simulation of Stokes' first problem. The flows around a stationary circular cylinder, two circular cylinders in tandem and an oscillating circular cylinder are simulated using the present method. The results are compared with other numerical and experimental results to assess the capability of the present method in handling complex as well as moving boundaries.

2. BGK scheme with external force term

The BGK scheme used in this paper is based on the numerical scheme proposed in Tian et al. [22]. Some construction strategies are complemented to apply this scheme to a 2D Cartesian mesh with quadtree refinement as shown in Fig. 1. Due to the complex procedure of the BGK scheme, here we just describe the solving process briefly and more details can be found in Tian et al. [22].

In this gas-kinetic scheme, the governing equations are the same as traditional finite-volume scheme

$$\frac{\partial}{\partial t}\int_{\Omega} \vec{U}d\Omega + \oint_{\vec{S}} \vec{F}\cdot d\vec{S} = \int_{\Omega} \vec{Q}\,d\Omega, \tag{1}$$

where Ω is the control volume and $\vec{S}$ is the volume surface. $\vec{U}$ is the macroscopic state vector, defined as