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Entropy-based artificial viscosity stabilization for non-equilibrium Grey Radiation-Hydrodynamics

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ABSTRACT

The entropy viscosity method is extended to the non-equilibrium Grey Radiation-Hydrodynamic equations. The method employs a viscous regularization to stabilize the numerical solution. The artificial viscosity coefficient is modulated by the entropy production and peaks at shock locations. The added dissipative terms are consistent with the entropy minimum principle. A new functional form of the entropy residual, suitable for the Radiation-Hydrodynamic equations, is derived. We demonstrate that the viscous regularization preserves the equilibrium diffusion limit. The equations are discretized with a standard Continuous Galerkin Finite Element Method and a fully implicit temporal integrator within the MOOSE multiphysics framework. The method of manufactured solutions is employed to demonstrate second-order accuracy in both the equilibrium diffusion and streaming limits. Several typical 1-D radiation-hydrodynamic test cases with shocks (from Mach 1.05 to Mach 50) are presented to establish the ability of the technique to capture and resolve shocks.

1. Introduction

Solving the radiation-hydrodynamic equations is a difficult task for multiple reasons. First, the characteristic time scales between the radiation and hydrodynamics are different by several orders of magnitude which often requires the radiation part to be solved implicitly to ensure stability. Second, as with any wave-dominated problems, high resolution schemes are needed to accurately resolve shocks. Third, high-order accuracy in time and space is challenging to achieve but some recent works provide examples of such results when solving either the Euler equations [1–4] or the radiation equation [5,6].

Substantial research efforts have focused on Riemann solvers for both the radiation and hydrodynamic equations. Balsara [7] developed a Riemann solver for the Radiation-Hydrodynamic equations by considering the frozen approximation that decouples the two physics components. However, such an approach may be questionable in the equilibrium diffusion limit. In this case, the coupling terms drive the physics and have to be accounted for. A *generalized* Riemann solver that accounts exactly for the relaxation terms was developed in [8]. Another approach assumes the strong equilibrium-diffusion limit (or frozen-in limit) in which radiation diffusion is negligible and the radiation simply advects at the material velocity [9]. In this limit, the radiation-hydrodynamic equations can be expressed in the form of the Euler equations with a radiation-modified equation of state (REOS). Edwards et al. [10] proposed a two-stage semi-implicit IMEX scheme to solve

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the Radiation-Hydrodynamic equations. A Riemann solver along with a flux limiter is used to resolve shocks and other waves. Their results show good agreement with semi-analytical solutions.

In this article we propose to solve the non-equilibrium Grey Radiation-Hydrodynamics (GRH) equations by stabilizing the numerical discretization using *the Entropy Viscosity Method* (EVM). This EVM, developed by Guermond et al. for hyperbolic systems of equations [2,3], consists in adding appropriate dissipative terms to the governing equations. The artificial viscosity coefficient in these terms is modulated by the local entropy production. The dissipative terms are devised to stabilize the numerical scheme and to remove the non-physical oscillations appearing at the shock locations. Since entropy production is peaked in shocks [11], the viscosity coefficient in the EVM is set proportional to the entropy production that is computed on the fly. In doing so, shocks can be detected and tracked and an adequate amount of artificial viscosity is added locally to stabilize the numerical scheme. The entropy viscosity method was shown to achieve high-order accuracy away from the shock regions, was successfully applied to nonlinear hyperbolic equations using various discretization methods (finite volume, continuous and discontinuous finite elements, spectral method), and yielded high-order accuracy on non-uniform meshes and complex geometries [3,12]. Because of the similarity between Euler equations and the radiation-hydrodynamic equations, it is conjectured that the entropy viscosity method may be a good candidate for resolving shocks occurring in radiation-hydrodynamic phenomena.

The 1-D non-equilibrium Grey Radiation-Hydrodynamic equations are recalled in Eqs. (1):

$$\partial_t(\rho) + \partial_x(\rho u) = 0 \tag{1a}$$

$$\partial_t(\rho u) + \partial_x\left(\rho u^2 + P + \frac{\epsilon}{3}\right) = 0 \tag{1b}$$

$$\partial_t(\rho E) + \partial_x[u(\rho E + P)] = -\frac{u}{3}\partial_x\epsilon - \sigma_a c\left(aT^4 - \epsilon\right) \tag{1c}$$

$$\partial_t\epsilon + \frac{4}{3}\partial_x(u\epsilon) = \frac{u}{3}\partial_x\epsilon + \partial_x\left(\frac{c}{3\sigma_t}\partial_x\epsilon\right) + \sigma_a c\left(aT^4 - \epsilon\right) \tag{1d}$$

where ρ, u, E, ϵ, P and T are the material density, material velocity, material specific total energy, radiation energy density, material pressure and temperature, respectively. The total and absorption cross sections, σ_t and σ_a, are either constant or given as a function of material density and temperature. The variables a and c are the radiation constant and the speed of light, respectively. The symbols ∂_t and ∂_x denote the temporal and spatial partial derivatives, respectively. The material temperature and pressure are computed with the ideal gas equation of state (IGEOS): $P = (\gamma - 1)C_v\rho T$ and $e = C_v T$, where $e = E - \frac{1}{2}u^2$ is the specific internal energy. The heat capacity C_v and the heat ratio coefficient γ are assumed constant.

The approach followed in this paper is similar to those of [7,13] where the relaxation and diffusion terms in the radiation and material energy equations are omitted in order to analyze only the hyperbolic parts of Eq. (1).

This paper is organized as follows. In Section 2, the entropy viscosity method is extended to the non-equilibrium Grey Radiation-Hydrodynamic equations. Details regarding the derivation of the adequate dissipative terms and definitions for the new viscosity coefficients are provided. Spatial and temporal discretization schemes are discussed in Section 3 along with the solution algorithm employed to solve the discretized equations. Numerical results are presented in Section 4 where the second-order accuracy of the scheme is demonstrated in both the equilibrium-diffusion and streaming limits, using the method of manufactured solutions. Then, several numerical test cases taken from the published literature [14] are provided; in these simulations, the Mach number varies from 1.05 to 50. Conclusions are presented in Section 5.

2. The entropy-based viscosity method applied to the Radiation-Hydrodynamic equations

In this section, we extend the entropy viscosity method [2,3,12] to the Radiation-Hydrodynamic equations in a staged process. First, the reader is guided through the main steps that lead to the derivation of the viscous regularization based on the entropy minimum principle [15]. Then, an asymptotic study is performed for the *regularized* GRH equations, i.e., *with* viscous dissipative terms present; we show that the equilibrium-diffusion limit is preserved for the regularized GRH equations. The frozen-in limit is also investigated and an equivalence is shown between the results presented in this paper and previously obtained results (for instance, the ones in [13]). Finally, a definition for the entropy viscosity coefficient is given along with the viscous regularization of the GRH equations.

2.1. Derivation of a viscous regularization for the non-equilibrium Grey Radiation-Hydrodynamic equations

We recall that the entropy viscosity method was developed for hyperbolic system of equations. However, the Radiation-Hydrodynamic equations are not strictly hyperbolic but several numerical techniques are based on the characteristics of their hyperbolic parts [7,13]. Following the same rationale, we first omit the relaxation and the diffusion terms in the material and radiation energy equations, Eq. (1c) and Eq. (1d); this yields the following hyperbolic system:

$$\partial_t(\rho) + \partial_x(\rho u) = 0 \tag{2a}$$

$$\partial_t(\rho u) + \partial_x\left(\rho u^2 + P + \frac{\epsilon}{3}\right) = 0 \tag{2b}$$