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# Anti-diffusion method for interface steepening in two-phase incompressible flow

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ABSTRACT

In this paper, we present a method for obtaining sharp interfaces in two-phase incompressible flows by an anti-diffusion correction, that is applicable in a straight-forward fashion for the improvement of two-phase flow solution schemes typically employed in practical applications. The underlying discretization is based on the volume-of-fluid (VOF) interface-capturing method on unstructured meshes. The key idea is to steepen the interface, independently of the underlying volume-fraction transport equation, by solving a diffusion equation with reverse time, i.e. an anti-diffusion equation, after each advection time step of the volume fraction. As the solution of the anti-diffusion equation requires regularization, a limiter based on the directional derivative is developed for calculating the gradient of the volume fraction. This limiter ensures the boundedness of the volume fraction. In order to control the amount of anti-diffusion introduced by the correction algorithm we propose a suitable stopping criterion for interface steepening. The formulation of the limiter and the algorithm for solving the anti-diffusion equation are applicable to 3-dimensional unstructured meshes. Validation computations are performed for passive advection of an interface, for 2-dimensional and 3-dimensional rising-bubbles, and for a rising drop in a periodically constricted channel. The results demonstrate that sharp interfaces can be recovered reliably. They show that the accuracy is similar to or even better than that of level-set methods using comparable discretizations for the flow and the level-set evolution. Also, we observe a good agreement with experimental results for the rising drop where proper interface evolution requires accurate mass conservation.



## 1. Introduction

The predictive simulation of two-phase flows is very important for process engineering and related technologies. For this purpose, various approaches have been proposed which can be differentiated into two main classes, interface-tracking methods and interface-capturing methods. With interface-tracking methods the location of the interface is explicitly represented, such as, e.g. front tracking methods [40,38] and marker methods [37,31]. Marker methods can locate efficiently the interface position by interface markers. However, they encounter difficulties for large interface deformations and topology changes, and require a special treatment of the interface marker distribution when the interface is stretched or compressed. Interface-capturing methods do not explicitly track but capture the location of the interface implicitly. Examples of interface-capturing methods include the level-set method [26,32,34] and the volume-of-fluid (VOF) method [12]. With the level-set method the interface is defined as the zero contour of a signed-distance function – the level-set function. The interface is sharp by definition, and steepness is maintained by recovering the signed distance property of the level-set function through

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reinitialization. From the level-set function the curvature of the interface and the surface tension can be calculated with high accuracy. However, a main drawback of the level-set method is lack of discrete conservation. With VOF [31,28] methods, two different phases are defined by the volume fraction of one phase within the other. The interface is represented by the transition region where the volume fraction ramps up from 0 to 1. The main advantage of VOF methods is the exact conservation of mass. One main drawback of VOF methods is that the interface cannot be located precisely, which leads to inaccuracies in calculating interface curvature and thus surface tension.

A sharp-interface representation can be obtained from the volume fraction by two approaches. With the first approach the interface is reconstructed before each advection step, and subsequently the flow is updated by propagating the reconstructed interface. Different interface-reconstruction schemes have been developed, such as simple line interface calculation (SLIC) [21], SOLA-VOF [12], and piecewise-linear interface construction (PLIC) as described by Scardovelli and Zaleski [31], Rider and Kothe [28] and references therein. Drawbacks of these VOF volume-tracking method are that the curvature of the reconstructed interface is not smooth, which leads to inaccuracies in the interface propagation step, and that the interface propagation can become unstable for very complex interfaces. Non-physical flows, commonly denoted as "flotsam" and "jetsam", can be created due to the errors induced by the particular volume-tracking algorithm [28]. With the second approach the volume fraction is advected with a special treatment to reduce the numerical diffusion. Examples of such VOF volume-capturing methods include a blending of a compressive scheme, satisfying a convection-boundedness criterion, with a higher order scheme for the advection step [39], and the introduction of an artificial-compression term in the advection equation [30]. It should be noted that the compressive discretization scheme requires special considerations to be applicable to unstructured meshes [39]. A recent overview of interface steepening schemes based on compressive flux formulations built into the volume-fraction transport is given by Cassidy et al. [7]. Other interface-capturing methods, among others, include those making use of a phase-field function [33] or a hyperbolic tangent function [42] to represent the interface profile and to compute the numerical flux for the fluid fraction function, as well as those employing a semi-Lagrangian conservative scheme [41] or a multi-integrated moment method [43].

For all methods, one prime criterion of accurate two-phase flow simulation is to maintain a sufficiently sharp interface throughout the simulation. How to obtain a sharp interface with interface-capturing methods has been investigated intensely in the past. Examples of models devised for recovering a sharp interface include the use of a limited downwind anti-diffusive flux correction [44] and the use of artificial compression as an intermediate step [22] within a level-set method. Unlike such previous approaches we propose a regularized anti-diffusion correction technique which can be used in a straight-forward fashion with any underlying VOF discretization scheme on structured or unstructured meshes. We demonstrate that this correction technique achieves a desired interface steepness reliably and efficiently. The main novelty is that the anti-diffusion correction algorithm is applied as post-processing step at each time step of the volume-fraction transport, independently of the VOF discretization scheme. The interface steepness lost during the VOF advection step is restored by solving an anti-diffusion equation explicitly in pseudo-time. Such postprocessing interface steepening mechanisms require a stopping criterion for defining the desired interface thickness while maintaining stability of the overall scheme. In other approaches, e.g. [22,23] a parameter is introduced relating the desired interface thickness to the grid. Here, we propose a different stopping criterion based on a discrete measure of the interface steepness. The advantage of this criterion is that it does not explicitly contain the grid-size as parameter, which would be cumbersome for unstructured meshes.

The governing equations and an overview of the solution procedure are presented in Section 2. The main focus of the paper, which is the formulation of the anti-diffusion equation and the solution algorithm, is presented in Section 3. Numerical cases and results are presented in Section 4. Finally, concluding remarks are given in Section 5.

## 2. Governing flow equations

The governing equations for unsteady, incompressible, viscous, immiscible two-phase flow are given by the continuity equation

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

and the Navier–Stokes equations

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \tau + \rho \mathbf{g} + \mathbf{f}_\sigma, \tag{2}$$

where $\mathbf{u}$ is the velocity, $\rho$ is the density, $t$ is time, $p$ is the pressure, $\boldsymbol{\tau}$ is the stress tensor, $\mathbf{g}$ is the gravitational acceleration, and $\mathbf{f}_\sigma$ is the force due to surface tension. Two different fluids are represented by the volume fraction $\alpha$ with $0 \leqslant \alpha \leqslant 1$, where $\alpha = 0$ refers to the first fluid, $\alpha = 1$ refers to the second fluid. $0 < \alpha < 1$ is the transitional region, i.e. the smeared discrete representation of the interface between the two fluids. The volume fraction is advected by the flow, resulting in the volume-fraction transport equation

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = 0. \tag{3}$$

Density $\rho$ and viscosity $\mu$ can be recovered from $\alpha$ by

$$\rho = \alpha \rho_1 + (1 - \alpha)\rho_2 \tag{4}$$