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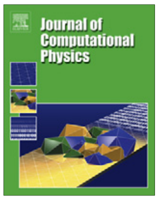

Time domain numerical modeling of wave propagation in 2D heterogeneous porous media

Guillaume Chiavassa [a,*], Bruno Lombard [b]

[a] M2P2, UMR 6181, CNRS, Ecole Centrale Marseille, Technopôle de Chateau-Gombert, 38 rue Frédéric Joliot-Curie, 13451 Marseille, France
[b] Laboratoire de Mécanique et d'Acoustique, UPR 7051 CNRS, 31 chemin Joseph Aiguier, 13402 Marseille, France

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ABSTRACT

This paper deals with the numerical modeling of wave propagation in porous media described by Biot's theory. The viscous efforts between the fluid and the elastic skeleton are assumed to be a linear function of the relative velocity, which is valid in the low-frequency range. The coexistence of propagating fast compressional wave and shear wave, and of a diffusive slow compressional wave, makes numerical modeling tricky. To avoid restrictions on the time step, the Biot's system is splitted into two parts: the propagative part is discretized by a fourth-order ADER scheme, while the diffusive part is solved analytically. Near the material interfaces, a space–time mesh refinement is implemented to capture the small spatial scales related to the slow compressional wave. The jump conditions along the interfaces are discretized by an immersed interface method. Numerical experiments and comparisons with exact solutions confirm the accuracy of the numerical modeling. The efficiency of the approach is illustrated by simulations of multiple scattering.

1. Introduction

The propagation of waves in porous media has crucial implications in many areas, such as the characterization of industrial foams, spongious bones and petroleum rocks. The most widely used model describing the propagation of mechanical waves in a saturated porous medium was proposed by Biot in 1956. A major achievement in Biot's theory was the prediction of a second (slow) compressional wave, besides the (fast) compressional wave and the shear wave classically propagated in elastic media.

Two regimes are distinguished, depending on the frequency of the waves. At frequencies smaller than a critical frequency f_c, the fluid flow inside the pores is of Poiseuille type, and the viscous efforts between the fluid and the solid depend linearly on the relative velocity. In this case, the slow compressional wave is almost static and highly attenuated [4]. An adequate modeling of this diffusive mode remains a major challenge in real applications. At frequencies greater than f_c, inertial effects begin to dominate the shear forces, resulting in an ideal flow profile except in the viscous boundary layer, and the slow wave propagates [5,32]. Experimental observations of the slow wave in the low-frequency range [36] and in the high-frequency range [10] have confirmed Biot's theory. In the current study, we focus on the low-frequency range.

Until the 1990s, Biot's equations were mainly studied in the harmonic regime. Various time-domain methods have been proposed since, based on finite differences [14,46,45], finite elements [47], discontinuous Galerkin methods [38], boundary elements [41], pseudospectral methods [7,8] and spectral element methods [34]. A recent review of computational

* Corresponding author. Tel.: +33 491 05 47 85.
E-mail addresses: guillaume.chiavassa@centrale-marseille.fr (G. Chiavassa), lombard@lma.cnrs-mrs.fr (B. Lombard).

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poroelasticity can be found in [9]. Nevertheless, none of the methods proposed in the low-frequency range give a complete answer to the following difficulties:

- the viscous effects greatly influence numerical stability, imposing a restrictive time step. In some physically relevant cases, computations cannot be carried out in a reasonable time;
- the wavelength of the slow compressional wave is much smaller than that of the other waves. Consequently, one faces the following alternative: either a coarse grid well-suited to the fast wave is chosen, and the slow wave is badly discretized; either a fine mesh is used, and the computational cost increases dramatically;
- maximum computational efficiency is obtained on a Cartesian grid; in counterpart, the interfaces are coarsely discretized, which yields spurious solutions. Alternatively, unstructured meshes adapted to the interfaces provide accurate description of geometries and jump conditions; however, the computational effort greatly increases, due to the cost of the mesh generation and to the CFL condition of stability.

The aim of the present study is to develop an efficient numerical strategy to remove these drawbacks. A time-splitting is used along with a fourth-order ADER scheme [42] to integrate Biot's equations. A flux-conserving space–time mesh refinement [3] is implemented around the interfaces to capture the slow compressional wave. Lastly, an immersed interface method [26,27] is developed to provide a subcell resolution of the interfaces and to accurately enforce the jump conditions between the different porous media. As illustrated by the simulations, the combination of these numerical methods highlights the importance of an accurate modeling of the slow wave.

This article, which generalizes a previous one-dimensional work [12], is organized as follows. Biot's model is briefly recalled in Section 2. The numerical methods are described in Section 3. Section 4 presents numerical experiments and comparisons with exact solutions. In Section 5, conclusions are drawn and future perspectives are suggested.

2. Physical modeling

2.1. Biot's model

Biot's model describes the propagation of mechanical waves in a porous medium consisting of a solid matrix saturated with fluid circulating freely through the pores [4,6,8,9]. It is assumed that

- the wavelengths are large compared with the diameter of the pores;
- the amplitudes of perturbations are small;
- the elastic and isotropic matrix is fully saturated by a single fluid phase.

This model relies on 10 physical parameters: the density ρ_f and the dynamic viscosity η of the fluid; the density ρ_s and the shear modulus μ of the elastic skeleton; the porosity $0 < \phi < 1$, the tortuosity $a \geqslant 1$, the absolute permeability κ, the Lamé coefficient λ_f and the two Biot's coefficients β and m of the saturated matrix. The unknowns are the elastic and acoustic displacements $\mathbf{u}_s$ and $\mathbf{u}_f$, the elastic stress tensor $\boldsymbol{\sigma}$, and the acoustic pressure p. In one hand, the constitutive laws are:

$$\begin{cases} \sigma = (\lambda_f \mathrm{tr}\varepsilon - \beta m \xi)\mathbf{I} + 2\mu\varepsilon, \\ p = m(-\beta \mathrm{tr}\varepsilon + \xi), \end{cases} \tag{1}$$

where $\mathbf{I}$ is the identity, ξ is the rate of fluid change, and ε is the strain tensor

$$\xi = -\nabla \cdot (\phi(\mathbf{u}_f - \mathbf{u}_s)), \quad \varepsilon = \frac{1}{2}\left(\nabla \mathbf{u}_s + \nabla \mathbf{u}_s^T\right). \tag{2}$$

The symmetry of σ in (1) implies compatibility conditions between spatial derivatives of ε, leading to the Beltrami–Michell equation [39,13]

$$\begin{aligned} &\frac{\partial^2 \sigma_{12}}{\partial x \partial y} = \theta_0 \frac{\partial^2 \sigma_{11}}{\partial x^2} + \theta_1 \frac{\partial^2 \sigma_{22}}{\partial x^2} + \theta_2 \frac{\partial^2 p}{\partial x^2} + \theta_1 \frac{\partial^2 \sigma_{11}}{\partial y^2} + \theta_0 \frac{\partial^2 \sigma_{22}}{\partial y^2} + \theta_2 \frac{\partial^2 p}{\partial y^2}, \\ &\theta_0 = -\frac{\lambda_0}{4(\lambda_0 + \mu)}, \quad \theta_1 = \frac{\lambda_0 + 2\mu}{4(\lambda_0 + \mu)}, \quad \theta_2 = \frac{\mu\beta}{2(\lambda_0 + \mu)}, \end{aligned} \tag{3}$$

where $\lambda_0 = \lambda_f - \beta^2 m$ is the Lamé coefficient of the dry matrix.

On the other hand, the conservation of momentum yields

$$\begin{cases} \rho \frac{\partial \mathbf{v}_s}{\partial t} + \rho_f \frac{\partial \mathbf{w}}{\partial t} = \nabla \sigma, \\ \rho_f \frac{\partial \mathbf{v}_s}{\partial t} + \rho_w \frac{\partial \mathbf{w}}{\partial t} + \frac{\eta}{\kappa}\mathbf{w} = -\nabla p, \end{cases} \tag{4}$$