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High order operator splitting methods based on an integral deferred correction framework

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ABSTRACT

Integral deferred correction (IDC) methods have been shown to be an efficient way to achieve arbitrary high order accuracy and possess good stability properties. In this paper, we construct high order operator splitting schemes using the IDC procedure to solve initial value problems (IVPs). We present analysis to show that the IDC methods can correct for both the splitting and numerical errors, lifting the order of accuracy by r with each correction, where r is the order of accuracy of the method used to solve the correction equation. We further apply this framework to solve partial differential equations (PDEs). Numerical examples in two dimensions of linear and nonlinear initial–boundary value problems are presented to demonstrate the performance of the proposed IDC approach.
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1. Introduction

In this paper we present high order operator splitting methods based on the integral deferred correction (IDC) mechanism. The methods are designed to leverage recent progress on parallel time stepping and offer a great deal of flexibility for computing the ordinary differential equations (ODEs). We focus on extending IDC theory to the case of splitting schemes on the IVP

$$u_t = f(t,u) = \sum_{\nu=1}^{\Lambda} f_\nu(t,u), \qquad u(0) = u_0, \qquad t \in [0,T], \tag{1.1}$$

and discuss the application in parabolic PDEs. Here, $u \in \mathbb{R}^n$ and $f(t,u) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$.

In the case that (1.1) arises from a method of lines discretization of time dependent PDEs which describe multi-physics problems, we encounter high dimensional computation. For these problems, the splitting methods can be applied to decouple the problems into simpler sub-problems. Therefore, the main advantages of operator splitting methods are problem simplification, dimension reduction, and lower computational cost. Two broad categories can classify many splitting methods: differential operator splitting [1,25,32,33] and algebraic splitting with the prominent example of the alternating direction implicit (ADI) method, which was first introduced in [9,7,30] for solving two dimensional heat equations. The main barrier in designing high order numerical methods based on the idea of splitting is the operator splitting error. To obtain high order accuracy via low order splitting method generally adds complexity to designing a scheme and stability

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analysis [14,27,18,26,34,12]. A recent work in [3] utilizes the spectral deferred correction (SDC) procedure to the advection–diffusion–reaction system in one dimension in order to enhance the overall order of accuracy. However, this work does not contain a proof that the corrections raise the order of the method.

In [10], an SDC method is first proposed as a new variation on the classical deferred correction methods [2]. The key idea is to recast the error equation such that the residual appears in the error equation in integral form instead of differential form, which greatly stabilizes the method. It is proposed as a framework to generate arbitrarily high order methods. This family of methods use Gaussian quadrature nodes in the correction to the defect or error, hence the method can achieve a maximal order of $2(M-1)$ on M grid points with $2(M-2)$ corrections. This main feature of the SDC method made it popular and extensive investigation can be found in [10,28,21,23,22,15,16,24]. Following this line of approach, the IDC methods are introduced in [6,5,4]. High order explicit and implicit Runge–Kutta (RK) integrators in both the prediction and correction steps (IDC-RK) are developed by utilizing uniform quadrature nodes for computing the residual. In [6,5], it is established that using explicit RK methods of order r in the correction step results in r higher degrees of accuracy with each successive correction step, but only if uniform nodes are used instead of the Gaussian quadrature nodes of SDC. It is shown in [5] that the new methods produced by the IDC procedure are yet again RK methods. It is also demonstrated that, for the same order, IDC-RK methods possess better stability properties than the equivalent SDC methods. Furthermore, for explicit methods, each correction of IDC or SDC increases the region of absolute stability. Similar results are generalized to arbitrary order implicit and additive RK methods in [4]. Generally, for *implicit* methods based on IDC and SDC, the stability region becomes smaller when more correction steps are employed. It is believed that this is due to the numerical approximation of the residual integral. The primary purpose of this work is to apply the IDC methods to the low order operator splitting methods in order to obtain higher order accuracy.

The paper is organized as follows. In Section 2, we briefly review several classical operator splitting methods and show how these methods can be cast as additive RK (ARK) methods. In Section 3 we formulate the IDC methodology for application to operator splitting schemes. In Section 4, we prove that IDC methods can correct for both the splitting and numerical errors of ODEs, giving r higher degrees of accuracy with each correction, where r is the order of the method used in the correction steps. In Section 5, as an interesting example, we will show how to use integral deferred correction for operator splitting (IDC-OS) schemes as a temporal discretization when solving PDEs. In Section 6 we carry out numerical simulations based on IDC methods for both linear and non-linear parabolic equations, and demonstrate that the new framework can achieve high order accuracy in time. In Section 7 we conclude the paper and discuss future work. We note that both the parallel time stepping version of IDC and the work presented in this paper are likely to benefit from the work in [20], and will be the subject of further investigation.

2. Operator splitting schemes for ODEs

In this section, we review several splitting methods which will serve as the base solver in the IDC framework. For differential operator splitting, such as Lie–Trotter splitting and Strang splitting, which happens at continuous level, we will apply appropriate numerical methods to the sub-problems and refer the whole approach as the discrete form of differential splitting. For both the differential splitting and algebraic splitting, we will show that each of the numerical schemes can be written as an ARK method. This insight is the first step required to apply the IDC methodology [4] to operator splitting schemes, which is the primary purpose of the present work.

2.1. Review of ARK methods

For IVP (1.1), when different p-stage RK integrators are applied to each operator L_ν, the entire numerical method is called an ARK method. If we define the numerical solution after n time steps as υ^n, which is an approximation to the exact solution $u(t_n)$, then one step of a p-stage ARK method is given by

$$\upsilon^{n+1} = \upsilon^n + \Delta t \sum_{\nu=1}^{\Lambda} \sum_{i=1}^{p} b_i^{[\nu]} f_\nu(t_n + c_i^{[\nu]}\Delta t, \tilde{\upsilon}_i), \tag{2.1}$$

with

$$\tilde{\upsilon}_i = \upsilon^n + \Delta t \sum_{\nu=1}^{\Lambda} \sum_{j=1}^{p} a_{ij}^{[\nu]} f_\nu(t_n + c_j^{[\nu]}\Delta t, \tilde{\upsilon}_j) \tag{2.2}$$

and $\Delta t = t_{n+1} - t_n$. An ARK method is succinctly identified by its Butcher tableau, as is demonstrated in Table 2.1.

In the following sections, we will explicitly write out the Butcher tableau for each operator splitting scheme and conclude that each of the operator splitting schemes considered in this work is indeed a form of ARK method.

2.2. Lie–Trotter splitting

We describe Lie–Trotter splitting for (1.1) in the case of $\Lambda = 2$ in the right hand side functions. We consider a single interval $[t_n, t_{n+1}]$. With first order Lie–Trotter splitting, (1.1) can be solved by two sub-problems:

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