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A conservative lattice Boltzmann model for the volume-averaged Navier–Stokes equations based on a novel collision operator

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ABSTRACT

The volume-averaged Navier–Stokes (VANS) equations are at the basis of numerous models used to investigate flows in porous media or systems containing multiple phases, one of which is made of solid particles. Although they are traditionally solved using the finite volume, finite difference or finite element method, the lattice Boltzmann method is an interesting alternative solver for these equations since it is explicit and highly parallelizable. In this work, we first show that the most common implementation of the VANS equations in the LBM, based on a redefined collision operator, is not valid in the case of spatially varying void fractions. This is illustrated through five test cases designed using the so-called method of manufactured solutions. We then present an LBM scheme for these equations based on a novel collision operator. Using the Chapman–Enskog expansion and the same five test cases, we show that this scheme is second-order accurate, explicit and stable for large void fraction gradients.

1. Introduction

Multiphase flows play a critical role in numerous key unit operations in the process industry such as mixing [1], transport [2] and fluidization [3]. Due to their complexity, they are often the bottleneck in the design and the operation of these units. Although the experimental study of these systems has led to a better understanding of their behavior, numerical simulation has proved an efficient and complementary tool to gain a deeper knowledge of the underlying flows.

Due to the steady increase of computational power, the last decades have witnessed the development of numerous numerical models that are capable of resolving multiphase flows with various length and time scales [4]. Among these, the two-fluid model [5,6], the combination of classical CFD approaches and the discrete element method (DEM) dubbed CFD-DEM [7], and the multiphase particle-in-cell (MP-PIC) method [8] have been used extensively to study, in particular, solid–fluid flows such as those in solid–liquid mixing [9], fluidized beds and pneumatic transport [3]. In such cases, these methods all have in common that they are based on the solution of a volume-averaged form of the Navier–Stokes (VANS) equations for either the two phases (two-fluid model) or for the fluid only (CFD-DEM and MP-PIC). The VANS equations have also been used extensively in the study of porous media, in which the porosity is a function of space [10].

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Traditionally, the VANS equations have been solved using classical numerical methods such as the finite volume method [11–14], the finite difference method [15] or the finite element method [10]. However, alternative numerical approaches have been proposed in recent years in the hope of increasing the versatility and the computational speed of the standard methods. These models are based on the use of smoothed particle dynamics (SPH) [16] or the lattice Boltzmann method (LBM) [17–20].

Among the last two numerical paradigms, the lattice Boltzmann method is an appealing candidate for the solution of the VANS equations. Indeed, this method is explicit and highly parallelizable, making it the ideal fluid solver in models such as CFD-DEM, which is generally computationally intensive because it requires small CFD time steps to ensure the stability of the coupling between the two phases. However, the solution of the VANS equations using the LBM requires a modified scheme to take into account the void fraction. The schemes that have been proposed in the literature can be grouped into two main categories. The first kind is based on a reformulation of the collision operator and an additional term to recover a pressure gradient that is independent of the void fraction [18–20]. In the present work, this type of model is referred to as pressure-correction LBM-VANS scheme. The second kind is based on a non-conservative formulation of the VANS equations and uses the classical LBM scheme along with mass and momentum source terms to recover the VANS equations [17].

As will be demonstrated in this paper, the pressure correction schemes are generally inadequate, even in the case of small void fraction gradients, due to their lack of robustness and accuracy. On the other hand, the non-conservative schemes require the use of mass source terms for which the implementation in the LBM is much more complex, requiring the solution of matrix systems and local sub-iterations. Furthermore, the expected second-order convergence of these two types of schemes has not been verified for non-trivial test cases in which the velocity and the volume fraction vary in space. This can be explained, at least in part, by the lack of non-trivial analytical solutions for the VANS equations.

Recently, Blais and Bertrand [21] have shown that the method of manufactured solutions (MMS) can be used to design complex test cases for the VANS equations, for which the convergence analysis of a solver can be carried out. They applied it successfully for the verification of the VANS equations within the CFDEM framework [22], which is based on the finite volume library Open∇FOAM [23] and DEM code LIGGGHTS [24,25].

In this work, we briefly present the VANS equations and recall the pressure-correction LBM-VANS scheme that has been proposed in the literature. Then, we explain how the method of manufactured solutions can be used to design analytical solutions for these equations. We show by means of five test cases that this pressure-correction LBM-VANS scheme suffers from instabilities, notably in situations where the fluid is static (no-flow tests). We then introduce a new LBM-VANS scheme that relies on a new collision operator originating from the so-called immiscible multiphase lattice Boltzmann method [26]. This model is analyzed theoretically using a Chapman–Enskog expansion before it is verified using the same five test cases. We show that this new LBM-based model is second-order accurate and discuss its robustness.

2. Volume-averaged Navier–Stokes equations

A number of forms of the VANS equations have been proposed in the literature for multiphase flows. The main differences between these forms relate to the treatment of the interphase coupling and the expression for the stress tensor, as thoroughly discussed by Zhou et al. [11] for the two-fluid and the CFD-DEM models.

In this work, we consider without loss of generality the so-called form A of the VANS equations, which is based on local averaging. We refer to the book by Gidaspow [6] for an in-depth description of the origin of the model. The form A of the VANS equations will be simply referred to as the VANS equations in the remainder of this work.

The incompressible VANS equations are:

$$\frac{\partial \epsilon_f}{\partial t} + \nabla \cdot \left(\epsilon_f \boldsymbol{u}\right) = 0 \tag{1}$$

$$\frac{\partial \left(\rho_f \epsilon_f \boldsymbol{u}\right)}{\partial t} + \nabla \cdot \left(\rho_f \epsilon_f \boldsymbol{u} \otimes \boldsymbol{u}\right) = -\epsilon_f \nabla p + \nabla \cdot \boldsymbol{\tau} + \boldsymbol{F} \tag{2}$$

where ϵ_f is the void fraction, ρ_f the density of the fluid, p the pressure, $\boldsymbol{u}$ the velocity and $\boldsymbol{F}$ a momentum source term. The viscous stress tensor, $\boldsymbol{\tau}$, is defined as [7]:

$$\boldsymbol{\tau} = \mu \epsilon_f \left((\nabla \boldsymbol{u}) + (\nabla \boldsymbol{u})^T - \frac{2}{3} (\nabla \cdot \boldsymbol{u}) \boldsymbol{\delta}_k \right) \tag{3}$$

where μ is the dynamic viscosity and $\boldsymbol{\delta}_k$ the identity tensor.

It is important to note that the velocity and void fraction resulting from these equations are not individually divergence free, which means that all terms of the stress tensor are *a priori* non-zero.

3. Lattice Boltzmann method

The Lattice Boltzmann Method (LBM) is based on the kinetic theory of gas and comes from the discretization in space, velocity and time of the Boltzmann equation. In fact, the LBM may be interpreted as the projection of the velocity space of the Boltzmann equation onto an isotropic orthonormal Hermite polynomial basis [27]. Consequently, the lattice Boltzmann