Journal of Computational Physics 294 (2015) 363–381

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An unstructured-mesh atmospheric model for nonhydrostatic dynamics: Towards optimal mesh resolution

Joanna Szmelter [a,*], Zhao Zhang [b], Piotr K. Smolarkiewicz [c]

[a] Loughborough University, Leicestershire, LE11 3TU, UK
[b] Heriot-Watt University, Edinburgh, EH14 4AS, UK
[c] European Centre For Medium Range Weather Forecasts, Reading, RG2 9AX, UK

ARTICLE INFO

Article history:
Received 16 September 2014
Received in revised form 22 March 2015
Accepted 30 March 2015
Available online 3 April 2015

Keywords:
Unstructured mesh atmospheric models
Nonoscillatory forward-in-time schemes
Anelastic equations
Orographic flows
Critical layer

ABSTRACT

The paper advances the limited-area anelastic model (Smolarkiewicz et al. (2013) [45]) for investigation of nonhydrostatic dynamics in mesoscale atmospheric flows. New developments include the extension to a tetrahedral-based median-dual option for unstructured meshes and a static mesh adaptivity technique using an error indicator based on inherent properties of the Multidimensional Positive Definite Advection Transport Algorithm (MPDATA). The model employs semi-implicit nonoscillatory forward-in-time integrators for soundproof PDEs, built on MPDATA and a robust non-symmetric Krylov-subspace elliptic solver. Finite-volume spatial discretisation adopts an edge-based data structure. Simulations of stratified orographic flows and the associated gravity-wave phenomena in media with uniform and variable dispersive properties verify the advancement and demonstrate the potential of heterogeneous anisotropic discretisation with large variation in spatial resolution for study of complex stratified flows that can be computationally unattainable with regular grids.

1. Introduction

Historically, atmospheric models across scales and foci of interests were dominated by finite-difference and spectral-transform methods for spatial discretisation of their governing PDEs. In particular, finite-difference methods operating on regular rectangular grids have prevailed in small and mesoscale models for research of cloud processes and flows over complex terrain [36], with terrain fitted grids mimicked by continuous mappings [13] and horizontal resolution refinement delegated to nested grids [6]. These techniques are still standard in computational studies from planetary boundary layer to regional climate [48,12].

Pioneered by the OMEGA model [1] for forecasting high-impact weather, air quality, and environmental hazard, there has been a growing interest in modelling atmospheric flows on unstructured meshes and utilising flexible mesh adaptivity. Interest in unstructured meshing per se dates back to the nineteen sixties [58], in the context of homogeneous discretisation for global flows. However, a broader research of flexible mesh adaptivity is relatively new — cf. the collection of papers [29] and seminars [17] — and adaptive-mesh atmospheric models have not yet achieved the maturity and acceptance of

---

* Corresponding author.
E-mail address: j.szmelter@lboro.ac.uk (J. Szmelter).

http://dx.doi.org/10.1016/j.jcp.2015.03.054

structured-grid models, commonly used in research and operations.[1] Fundamental balances and wave propagation underlaying atmospheric dynamics pose new challenges to anisotropic heterogeneous discretisation and flexible mesh adaptivity, largely developed in the engineering community for neutrally stratified non-rotating flows [59]; for a substantive discussion see [10].

However, unstructured-meshes and mesh adaptivity offer flexibility unmatched by the established techniques operating on regular grids. Even though the latter enable computationally efficient static and dynamic mesh adaptivity via continuous mappings [31,56,21,3,4], their rigid connectivity imposes stringent constraints on adapted grids. Flexible unstructured meshes relax the constraints and offer alternative means for optimising variable resolution required for improved representation of complex physical processes in atmospheric flows. Generally, such flows evince the multiplicity of scales ranging from a fraction of a millimetre where dissipation occurs, to tens of thousands of kilometres where planetary weather and climate take place. For some atmospheric processes, that are still insufficiently understood in spite of their relevance to weather conditions in populated areas, flexible mesh adaptivity can improve solution realizability, and thus cognition and predictability, attainable with available computational resources. Examples include weather in long winding valleys and mountainous areas, onset and evolution of radiation fog or stratiform clouds, precipitation or extreme events forecasting. This paper advances the three-dimensional, nonhydrostatic, limited-area model designed for flexible fully unstructured meshes [45] by introducing its implementation on tetrahedral based median-dual meshes that enable a wide range of mesh refinement strategies.

The current work extends our earlier developments aiming at generalisation of the nonoscillatory forward-in-time (NFT) integrators to unstructured meshes. The NFT integrators were first proposed for finite-difference atmospheric models [39] and were proven for a range of computational studies from ground-water flows [19], through all scale atmospheric dynamics [46], to magnetic cycles in global stellar convection [14]. The roots of the NFT schemes for unstructured meshes are in the derivation from first principles of the finite-volume multidimensional positive definite advection transport algorithm (MPDATA) together with the stability and convexity theory, convergence analysis, and several MPDATA options [41,42].[2] Further developments included an unstructured-mesh finite-volume NFT framework for gas dynamics with a selection of techniques for mesh adaptivity [50,43] originally developed for engineering flows [59,49,51]. The applicability of NFT methods to gas dynamics was demonstrated through verifications against benchmarks for all-speed aerodynamic flows with favourable comparisons to established solutions (theoretical and numerical) and convergence studies. Subsequent developments of the unstructured meshes based NFT integrators focused on meteorological applications producing models for simulating idealised hydrostatic dynamics of the planetary atmosphere [52], reduced 2D soundproof models for simulation of nonhydrostatic gravity-wave dynamics [53,44] and their consequent generalisation to 3D mesoscale modelling of nonhydrostatic dynamics [45] — all supported with extensive verification studies.

The underlying concept of the numerical model used in the current study is presented in detail in [45]. In the model, all dependent variables are co-located, benefiting memory and communication requirements compared to staggered arrangements. This also facilitates implicit representation of buoyant modes. Generally, NFT labels a class of second-order-accurate two-time-level schemes (of the Cauchy–Kowalewski type, cf. Section 19 in [54]) for integrating fluid PDEs that are built on nonlinear advection schemes such as MPDATA. The MPDATA schemes control numerical oscillations in the sense of high-resolution methods [33], and this assures the nonlinear stability for the co-located arrangement of dependent variables (cf. Appendix A1 in [42]). Furthermore, in the NFT solver, MPDATA provides implicit turbulence modelling capability to the full set of equations. The implicit large eddy simulation (ILES) properties of MPDATA-based high-Reynolds-number solvers were widely documented in the context of structured grids [24,25,7,26,55]. More recently, they were also demonstrated for unstructured meshes [53,44,45] and verified against published data and LES/ILES results generated with established codes in [45]. For unstructured meshes with large variation in spatial resolution, the ILES capability is especially practical, as it circumvents evaluation of explicit subgrid-scale models and construction of commutative filters for LES [27], in particular, and does not impede the stability of calculations.

The preceding works on the nonhydrostatic unstructured-mesh NFT models [53,44,45] verified the excellent accuracy of the finite-volume approach, using benchmarks from both analytic and laboratory results, and comparing unstructured-mesh results to the corresponding results obtained with the structured-grid EULAG model [32]. The previously studied physical problems representative of mesoscale dynamics included nonhydrostatic mountain waves at weak and strong background stratification (with linear and nonlinear flow responses, respectively), amplification and breaking of deep stratospheric gravity waves, low Froude number flows past steep isolated 3D mountain, and evolution of convective planetary boundary layer. Here, we advance simulation of flows past complex terrain and associated gravity-wave phenomena. In particular, we consider the special problem of a stratified flow past a two-scale slab-symmetric mountain — known to bifurcate into a qualitatively incorrect solution at coarse resolutions — to demonstrate the capabilities of unstructured meshes and static adaptive mesh refinement. Notably, an a posteriori error indicator that drives the mesh refinement arises from the inherent properties of MPDATA [50]. We also revisit 3D low Froude number flows to verify the accuracy of tetrahedral discretisation.

---

[1] According to the Thomson Reuters Web of Science, among nearly 900 papers published since 1985 on the topic of “mesh adaptivity” less than 1.5% fall into the category “meteorology atmospheric sciences”.

[2] The potential of MPDATA for unstructured mesh modelling was recognised much earlier: first in context of mantle convection models [5], and then in the area of weather and environmental modelling [1]; however the derivation in [42] is the first to encompass error compensating terms required for unstructured meshes.