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Tau method for the numerical solution of a fuzzy fractional kinetic model and its application to the oil palm frond as a promising source of xylose

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ABSTRACT

The Oil Palm Frond (a lignocellulosic material) is a high-yielding energy crop that can be utilized as a promising source of xylose. It holds the potential as a feedstock for bioethanol production due to being free and inexpensive in terms of collection, storage and cropping practices. The aim of the paper is to calculate the concentration and yield of xylose from the acid hydrolysis of the Oil Palm Frond through a fuzzy fractional kinetic model. The approximate solution of the derived fuzzy fractional model is achieved by using a tau method based on the fuzzy operational matrix of the generalized Laguerre polynomials. The results validate the effectiveness and applicability of the proposed solution method for solving this type of fuzzy kinetic model.

1. Introduction

One of the ways for preventing the global warming is utilizing biofuels derived from lignocellulosic material instead of using fossil fuels [1]. We recall that the oil palm is a multipurpose plant and one of the largest sources of the lignocellulosic biomass for value-added industries that produces massive amounts of agricultural co-products, e.g. empty fruit bunch fibers, fronds, trunks and shells [2].

The utilization of natural lignocellulosic materials to produce environmentally friendly biofuel has been regarded as an alternative way to cope with environmental pollution problems [3]. The chemical composition analysis reveals that Oil Palm Frond (OPF) biomass consists of hemicellulose, which is a sugar polysaccharide, made mainly of C_5 sugar, i.e., xylose and xylooligosaccharides. Besides that, the other byproducts of OPF biomass include cellulose, arabinose, glucose, acetic acid and furfural.

Hydrolysis is known as the most effective method to break down polymer chains of cellulose and hemicellulose into their sugar monomers. The process is performed in the presence of water to break of the glycosidic bond within a polysaccharide chain [4]. Acid hydrolysis reveals oligomers and monosaccharide that have already been derived as a homogeneous reaction

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model in which acid catalyzes the breakdown of the hemicellulose to xylose followed by the breakdown of xylose to furfural [5]. Saeman's kinetics was adapted to depict the hydrolysis of hemicellulose and the formation of byproducts at high temperatures by several scientists [6,7].

In this study, the kinetics of diluted acid hydrolysis in OPF was determined by using the optimized condition which was obtained by 2%–6% H_2SO_4 at 120 °C. Our focus has been confined to finding the concentration of high yield xylose in the hemicellulose hydrolysis by introducing a fuzzy fractional kinetic model instead of Saeman's classic model [6]. The constant rates were determined as functions of temperature and sulfuric acid concentration.

We recall that the fractional calculus possesses a long history and it deals with various aspects of engineering and economic applications [8–10]. The number of mathematical models engaging fractional operators is still in progress. It has been discovered that various critical and significant applications can be gracefully modeled with the aid of the fractional derivatives. We mention that the fractional derivatives and integrals provide more precise models of the systems under consideration. The researchers have proved the existence of fractional calculus in anomalous diffusion [11], medicine [12], viscoelastic [13], random and disordered media [14], signal processing [15], and so on.

Generally, the majority of the fractional differential equations do not have exact solutions, as a result the approximate and numerical procedures [16,17] have to be utilized. Different numerical techniques to solve fractional differential equations have been offered, e.g. variational iteration method [18], homotopy perturbation method [19], waveform relaxation methods [20] and collocation method [21,22]. Expression of a function in terms of a series expansion exploiting orthogonal polynomials is an essential concept in the approximation theory and constitutes the foundation for solving differential equations [23]. Recently, a considerable attention was given to applying the orthogonal functions for solving fractional differential equations [24–28].

Although commonly the unknown parameters involved in the models are assumed to be constant over time, in a more adequate perspective of any phenomenon, some of them are not constant and depend implicitly on several factors. We recall that some of such factors usually do not appear explicitly in the existing mathematical models because of the need for balance between modeling and numerical tractability and the lack of their exact knowledge. To deal with the uncertainty in the parameters, the stochastic approach is commonly used. The commitment is not to assume constant parameters, and assume that stochastic behavior implies knowledge of the probabilistic information of the system components. The derivation of such information is not completely reliable and does not have a high degree of accuracy. Fuzzy differential equations of integer and fractional orders have also been suggested as a way to model uncertain and/or incompletely specified systems [29,30]. In recent years, many researches were interested in the theory of fuzzy differential equations [31,32]. In addition the fuzzy fractional differential equations (FFDEs) or systems have been considered under different interpretations in some recent papers [33,34,34,36–41].

In this manuscript, we deal with the application of fractional differential equations to model the kinetic's behavior of the diluted acid hydrolysis in OPF. We determine the concentration of xylose in this process under uncertainty in parameters based on the fuzzy Caputo differentiability introduced in [37,39]. For our case, the fuzzy fractional differential equations (FFDEs) seem to be better suited to model practical situations under uncertainty and imprecision than formulations by means of classical ordinary differential equations. Moreover, we have simultaneously considered the Stacking theorem [42] to state all types of fractional Caputo differentiability. However, since the influence of some uncertain factors on the evolution of fractional dynamical system takes place very often, this apparatus would be more and more comprehensive in the case when it allows for consideration the fuzziness. Motivated by the results in [43–45], a promising operational matrix algorithm is presented to derive the fuzzy approximate solutions of FFDEs based on the generalized Laguerre functions defined over $[0, 1]$. The distinguished property of this method is that it gives an easy and simple algorithm in converting FFDEs to a system of fuzzy linear algebraic equations. This technique has various profits such as being non-differentiable, non-integral and comfortably implemented on a computer, because its structure is dependent on the matrix operations only.

The manuscript is structured as follows: Section 2 is assigned to review the significant notations and definitions related to the fuzzy settings theory and fuzzy differential equations of integer and fractional order. In Section 3, we illustrate the process which lead us to derive the classical kinetic equation based on the chemical reaction mentioned in Section 1. Also, the fuzzy fractional kinetic equation is presented in this section. Section 4 is allotted to developing a practical algorithm to approximate the reachable sets for the solution of the given fuzzy fractional kinetic equations (FFKEs). Some computational implementation details are analyzed in Section 5. The conclusions are given in Section 6.

2. Basic definitions

In this section, the necessary mathematical foundations related to the fuzzy settings theory and fuzzy differential equations are reviewed which will be used throughout the paper.

2.1. Fuzzy sets and spaces

The definitions presented in this section are provided from [42,46], unless specially indicated.

Definition 2.1. Let u be a fuzzy set in $\mathbb{R}$. u represents a fuzzy number if: