Journal of Computational Physics 293 (2015) 142–156

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A spectral tau algorithm based on Jacobi operational matrix for numerical solution of time fractional diffusion-wave equations

A.H. Bhrawy [a,b], E.H. Doha [c], D. Baleanu [d,e,f,*], S.S. Ezz-Eldien [g]

[a] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia
[b] Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt
[c] Department of Mathematics, Faculty of Science, Cairo University, Giza, Egypt
[d] Department of Chemical and Materials Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia
[e] Department of Mathematics and Computer Sciences, Cankaya University, Ankara, Turkey
[f] Institute of Space Sciences, Magurele-Bucharest, R 76900, Romania
[g] Department of Basic Science, Institute of Information Technology, Modern Academy, Cairo, Egypt

ARTICLE INFO

Article history:
Received 11 January 2014
Received in revised form 7 March 2014
Accepted 23 March 2014
Available online 3 April 2014

Keywords:
Fractional diffusion-wave equations
Tau method
Shifted Jacobi polynomials
Operational matrix
Caputo derivative

ABSTRACT

In this paper, an efficient and accurate spectral numerical method is presented for solving second-, fourth-order fractional diffusion-wave equations and fractional wave equations with damping. The proposed method is based on Jacobi tau spectral procedure together with the Jacobi operational matrix for fractional integrals, described in the Riemann–Liouville sense. The main characteristic behind this approach is to reduce such problems to those of solving systems of algebraic equations in the unknown expansion coefficients of the sought-for spectral approximations. The validity and effectiveness of the method are demonstrated by solving five numerical examples. Numerical examples are presented in the form of tables and graphs to make comparisons with the results obtained by other methods and with the exact solutions more easier.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Due to the accuracy of ordinary and partial fractional differential equations in modeling various engineering and physical phenomena such as bioengineering [25], solid mechanics [36], anomalous transport [28], continuum and statistical mechanics [26], fluid-dynamic [16], economics [1], nonlinear oscillation of earthquakes [15], colored noise [27] and many other problems [33], fractional calculus has become the focus of many researchers in recent years.

The time-fractional diffusion-wave equation, which is a mathematical model of a wide class of important physical phenomena, is a linear integro-partial fractional differential equation that obtained from the classical diffusion-wave equation by replacing the second-order time derivative term by a fractional derivative of order ν, $1 < \nu \leqslant 2$. Many of the universal mechanical, acoustic and electromagnetic responses may be described accurately by the time-fractional diffusion-wave equation, see [31,32]. The fourth-order space derivative is arising in the wave propagation in beams and modeling formation of grooves on a flat surface, thus considerable attention has been devoted to fourth-order fractional diffusion-wave equation and its applications, see [33].

* Corresponding author.

http://dx.doi.org/10.1016/j.jcp.2014.03.039
0021-9991/© 2014 Elsevier Inc. All rights reserved.

Several numerical methods have been proposed in the last few years for solving such equations. In [6,7], Cui presented high-order compact finite difference methods to solve the fractional diffusion and time fractional diffusion equation. A compact difference scheme [17,12] is proposed for solving second- and fourth-order fractional diffusion-wave equations. Chen et al. [5] proposed and developed the method of separation of variables with constructing the implicit difference scheme for fractional diffusion-wave equation with damping. Recently, Hu and Zhang [18] proposed the finite difference scheme for approximating the solution of the fourth-order fractional diffusion-wave equation. An explicit analytical solution is proposed in [14] for space–time fractional wave equation. In [8], Darzi et al. solved fractional diffusion-wave equations, by using the Sumudu transform method. Some other numerical methods are introduced for solving fractional differential equations with sub-diffusion and super-diffusion, see [13,21,22,4,20,35,40].

Spectral methods are a class of important tools for obtaining the numerical solutions of fractional differential equations. They have excellent error properties and they offer exponential rates of convergence for smooth problems [3,38,39]. Considerable attention has been devoted to the use of orthogonal polynomials aiming to deal with various fractional differential equations, by reducing them to systems of algebraic equations that greatly simplify obtaining their solutions. Moreover, the operational matrices of fractional derivatives are derived and used together with spectral methods based on orthogonal polynomials for developing numerical approximations to fractional differential equations, see [9,10]. Recently, and with the help of operational matrices of fractional derivatives for orthogonal polynomials, the tau spectral method is also utilized to solve spectrally space-fractional diffusion equations, see [11,34,37].

In the current work, we investigate and develop some efficient spectral algorithms to solve spectrally second- and fourth-order fractional diffusion-wave equations and fractional diffusion-wave equation with damping. These algorithms are based principally on shifted Jacobi tau-spectral method combined with the generalized shifted Jacobi operational matrix of integrals.

The current article is organized as follows. In Section 2, the shifted Jacobi polynomials with their properties and their operational matrix to fractional integration are presented. In Sections 3, 4 and 5, the shifted Jacobi operational matrix of fractional integration is applied together with the shifted Jacobi tau spectral method to give numerical solutions for second- and fourth-order fractional diffusion-wave equations and fractional diffusion-wave equation with damping, respectively. In Section 6, several numerical examples and comparisons between our obtained numerical results and those of other methods are considered. Also a conclusion is given in Section 7.

2. Operational matrices of shifted Jacobi polynomials

Riemann–Liouville and Caputo fractional definitions are the two most used from all the other definitions of fractional calculus which have been introduced recently.

Definition 2.1. The integral of order $\nu \geqslant 0$ (fractional) according to Riemann–Liouville is given by

$$\begin{aligned} I^{\nu} f(x) &= \frac{1}{\Gamma(\nu)} \int_0^x (x-t)^{\nu-1} f(t)\, dt, \quad \nu > 0,\ x > 0, \\ I^{0} f(x) &= f(x), \end{aligned} \tag{2.1}$$

where

$$\Gamma(\nu) = \int_0^{\infty} x^{\nu-1} e^{-x}\, dx$$

is gamma function.

The operator J^{ν} satisfies the following properties

$$\begin{aligned} I^{\nu} I^{\mu} f(x) &= I^{\nu+\mu} f(x), \\ I^{\nu} I^{\mu} f(x) &= I^{\mu} I^{\nu} f(x), \\ I^{\nu} x^{\beta} &= \frac{\Gamma(\beta+1)}{\Gamma(\beta+1+\nu)} x^{\beta+\nu}. \end{aligned} \tag{2.2}$$

Definition 2.2. The Caputo fractional derivative of order ν is defined by

$$D^{\nu} f(x) = \frac{1}{\Gamma(m-\nu)} \int_0^x (x-t)^{m-\nu-1} \frac{d^m}{dt^m} f(t)\, dt, \quad m-1 < \nu \leqslant m,\ x > 0, \tag{2.3}$$

where m is the ceiling function of ν.

The operator D^{ν} satisfies the following properties

Download English Version:

https://daneshyari.com/en/article/519738

Download Persian Version:

https://daneshyari.com/article/519738

Daneshyari.com