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Variational image registration by a total fractional-order variation model ☆

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ABSTRACT

In this paper, a new framework of nonlocal deformation in non-rigid image registration is presented. It is well known that many non-rigid image registration techniques may lead to unsteady deformation (e.g. not one to one) if the dissimilarity between the reference and template images is too large. We present a novel variational framework of the total fractional-order variation to derive the underlying fractional Euler–Lagrange equations and a numerical implementation combining the semi-implicit update and conjugate gradients (CG) solution to solve the nonlinear systems. Numerical experiments show that the new registration not only produces accurate and smooth solutions but also allows for a large rigid alignment, the evaluations of the new model demonstrate substantial improvements in accuracy and robustness over the conventional image registration approaches.

1. Introduction

One of the most important tasks in computer vision and image processing is registration, aiming to find a geometrical transformation that aligns points in one view of an object with corresponding points in another view of that object or another object, i.e., realigns two images – the reference and template images. Nowadays, image registration has played an important role in different applications, such as remote sensing, medicine and computer vision. Especially in medical diagnosis [1–4], for example, the efficient implementation of the automating medical diagnosis with the aid of computers should depend on reliable registration methods.

In addition to simple parameter based methods [5], the optical flow based approach is an early variational method, aiming to recover the displacement field between two frames of a video sequence which are taken at different times at every voxel position, so local Taylor series approximations of the image signal and the partial derivatives with respect to the spatial and temporal coordinates are used to calculate the motion between two images.

During the last decades, to realize image registration, a great number of variational approaches in the purpose of minimizing the similarity measures have been proposed. The similarity measures are used to quantify the degree of similarity

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between intensity patterns within two images. Since the underling problem is in general ill-posed in the sense of Hadamard, therefore, how to effectively minimize the similarity measures becomes a fundamental task in image sciences.

Regularizing ensures that the resulting well-posed problem admits a solution. In Tikhonov framework, the cost energy functional minimized in the registration model is a combination of the image similarity and the regularizing penalty functional. On one hand the choice of an image similarity measure depends on the modality of the images to be registered [6], including single-modality and multi-modality methods. Single-modality methods tend to register images in the same modality acquired by the same scanner type [7,8], while multi-modality registration methods tend to register images acquired by different scanner types. Common examples of image similarity measures include cross-correlation, mutual information and sum of squared intensity differences (SSD) [7,8]. Mutual information and normalized mutual information are the two of most popular image similarity measures for registration of multimodality images, while cross-correlation and SSD are commonly used for registration of images in the same modality. Image registration algorithms can be also classified into intensity-based and feature-based [6]. Intensity-based methods compare intensity patterns in images via correlation metrics, while feature-based methods find correspondence between image features such as points, lines, and contours [6].

On the other hand, image registration algorithms can also be classified according to the transformation models being used to relate the template (target) image space to the reference image space. The first category of transformation models refers to linear/affine transformations, which include rotation, scaling, translation [7]. Linear transformations are global in nature, thus, they cannot model local geometric differences between images [6]. The second category of transformations allows 'elastic' or 'nonrigid' transformations. These transformations are capable of locally warping the template image to align with the reference image. Nonrigid transformations include radial basis functions [6], physical continuum models [8–12] and large deformation models (diffeomorphisms) [13]. In all cases, it is preferable to choose transformations that have physical meaning, but in some cases, the choice is made on the basis of convenient mathematical properties. However, large local and global deformations may occur and must be taken into account.

Over the last decade, it has been demonstrated that many systems in science and engineering can be modeled more accurately by employing fractional-order rather than integer-order derivatives [14–16], and many methods are developed to solve the fractional systems [17–22]. Not all of these results have been considered for imaging applications. Recently, there have been several works involving discrete forms of an α-order derivative proposed to tackle the image restoration problem [23–30] and the image inpainting problem [31]. However much fewer works employing partial fractional α-order derivatives are applied to the image registration problem. Melbourne et al. [32] used fractional differentiation (differentiation to non-integer order) to design new gradients of image intensities for enhancing image registration performance to directly register image gradients. Garvey et al. in [33] proposed a nonrigid registration algorithm that involves directly and rapidly solving a discretized fractional PDE modeling super-diffusive processes in nonrigid image registration. The proposed algorithm yields lower average deformation errors than standard diffusion-based registration through registration experiments on breast MR imagery with simulated biomechanical deformations. In [34], a regularization term based on fractional order derivatives is introduced but the problem is solved in the frequency domain of the minimizing energy functional via the Euler–Lagrange equations. In [35], medical image registration was studied in the domain of fractional Fourier transform. These earlier works have suggested and illustrated that fractional order derivatives may be effective regularizers for image registration applications.

The contributions of this work are the following

i). We propose a new nonlocal deformation model with the total fractional-order variation regularizer in non-rigid image registration in a continuous setting. Due to the nonlocal field theories of fractional derivative, the new registration can produce accurate and steady smooth deformation.
ii). We establish better and more rigorous theories for applications of the total fractional-order variation to image inverse problems. To apply the total fractional-order variation regularization with fractional order derivative to variational image inverse problems, we analyze properties of the total fractional-order variation and its fractional integration by parts formulas from variational principles. We derive the Euler–Lagrange equation in suitable function spaces.
iii). We present a new numerical scheme combining the semi-implicit update, discretization matrix approximation and CG iterative solution.

Our work will facilitate future applications of α-order variation based regularizer to other imaging problems where regularization is required.

The rest of the paper is organized as follows. Section 2 first reviews the basic image registration problem, the demons algorithm, several variational models about image registration, definitions and basic properties of the fractional order derivative which help us to understand the differences between integer and fractional derivatives. In Section 3 we first discuss definition and properties of the total fractional-order variation which generates TV regularization with integer derivative. Then a total fractional-order variation image registration model with nonlocal property is considered, we should discuss the derivation of Euler–Lagrange equation and boundary condition. Before ending this section the study of discretization of Euler–Lagrange equation and efficient numerical schemes are developed. Experimental results are shown in Section 4, and the paper is concluded with a summary in Section 5.