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Conservative integral form of the incompressible Navier–Stokes equations for a rapidly pitching airfoil

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ABSTRACT

This study provides a simple moving-grid scheme which is based on a modified conservative form of the incompressible Navier–Stokes equations for flow around a moving rigid body. The modified integral form is conservative and seeks the solution of the absolute velocity. This approach is different from previous conservative differential forms [1–3] whose reference frame is not inertial. Keeping the reference frame being inertial results in simpler mathematical derivation to the governing equation which includes one dyadic product of velocity vectors in the convective term, whereas the previous [2,3] needs to obtain the time derivative with respect to non-inertial frames causing an additional dyadic product in the convective term. The scheme is implemented in a second-order accurate Navier–Stokes solver and maintains the order of the accuracy. After this verification, the scheme is validated for a pitching airfoil with very high frequencies. The simulation results match very well with the experimental results [4,5], including vorticity fields and a net thrust force. This airfoil simulation also provides detailed vortical structures near the trailing edge and time-evolving aerodynamic forces that are used to investigate the mechanism of the thrust force generation and the effects of the trailing edge shape. The developed moving-grid scheme demonstrates its validity for a rapid oscillating motion.

1. Introduction

Numerically representing unsteady flow due to motion of a moving rigid body with possible flow-structure interactions is challenging, and a number of approaches have been pursued. Among them are immersed boundary method [6], adaptive remeshing [7], overlapping meshes [8], and conservative forms of the Navier–Stokes equations in non-inertial reference frames [1–3]. The first three methods require additional computations in solving the governing Navier–Stokes equations: the immersed boundary method needs mass and momentum source terms for the immersed boundaries, the adaptive method needs to regenerate mesh along with the moving body, and the overlapping meshes need interpolation in the overlapped region. Instead, the conservative forms provide the governing equations for grids fixed to moving bodies so do not necessarily need the extra computations. The conservative forms avoid the non-conservative momentum source terms (e.g., Coriolis and angular acceleration terms in rotating frames) in the Navier–Stokes equations for a relative velocity in a non-inertial reference frame [2,6].

Warsi [1] derived a conservative form of the Navier–Stokes equations in non-inertial curvilinear coordinates. A similar form was used by Visbal [9] for a plunging airfoil. Beddhu et al. [2] simplified the conservative momentum equation (i.e.,

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Eq. 3.164 of [1]) to the vector form in non-inertial Cartesian coordinates (i.e., Eq. 2.14 of [2] for a self-gravitating, rotating body and Eq. 2.16 of [2] for a rotating frame in a gravitational field). Although Beddhu et al. [2], used the assumption of constant angular velocity for the Cartesian in the middle of their derivation, the final forms can be also used for time-dependent angular velocity.[1] The conservative form in a gravitational field (i.e., Eq. 2.16 of [2]) is applied for moving bodies by Kim and Choi [6]. Luo and Bewley [3] independently derived the conservative form using two different approaches (via the appropriate temporal differentiation in non-inertial curvilinear coordinates and, separately, the direct transformation from inertial Cartesian to non-inertial curvilinear coordinates).

The proposed moving-grid scheme follows the basic idea of the conservative forms for numerically stable solution. Similar to the previous conservative forms [1–3], the proposed conservative form seeks the solution of the absolute velocity (the non-conservative form seeks the relative velocity). Vectors and tensors are expressed in the non-inertial coordinates that are fixed to the moving body, which is another similarity.

Contrast to the previous [1–3], the proposed conservative form is based on a modified integral form of the incompressible Navier–Stokes equations for an arbitrarily moving control volume. The standard integral form (e.g., Eqs. 5.13.2 and 5.14.1 of [10]) is enough if computation domain is inertial and the positions of grid points are moving in the domain. In most cases and also here, however, the computation domain is fixed to the moving grid so the positions of grid points are invariant in time in the domain. In consequence, the integral form needs to be modified. Since the motion of the moving grid with respect to the inertial frame is already included in the convective term of the integral form, the modification should include the time-dependent orientation of the non-inertial coordinates with respect to the inertial frame.

This approach has several advantages over the previous[1–3]. (1) It does not need to convert the time derivative in the Navier–Stokes equations with respect to the inertial frame into that with respect to the non-inertial frame. In consequence, mathematical derivation for the modification of the standard integral form is simpler than that for the previous forms. This advantage is closely related to the followings. (2) The convective term maintains its form after the modification, whereas the convective terms in the previous (i.e., Eq. 2.16 of [2] and Eq. 28 of [3]) include an additional dyadic product of velocity vectors. In results, one convecting velocity is required for this approach whereas two velocities in the others. (3) The simple form of the governing equations, specifically the convective term, makes numerical implementation straightforward, otherwise further discretization methods are required for the additional dyadic product.

Details about the proposed moving-grid scheme are presented in Section 2 with its implementation in a Navier–Stokes solver CDP. The verification of this scheme is included in Section 3. The numerical scheme is used to simulate a high-frequency pitching airfoil in Section 4 which discusses detailed flow structures and the mechanism of the thrust force generation. Conclusions and future work are summarized in Section 5.

2. Numerics

The moving-grid scheme developed in this study modified the standard integral form of the incompressible Navier–Stokes equations for a control volume which is moving with respect to a stationary inertial reference frame. The modified integral form is conservative as is the standard form and is solved for the absolute velocity. This conservative form has several advantages to the previous [1–3], which is discussed in this section. For the simulations considered here, grids are rigidly moved with prescribed motions. Details of this scheme are described in Section 2.1, and then its implementation in a finite-volume Navier–Stokes solver CDP is discussed in Section 2.2.

2.1. A moving-grid scheme

In order to introduce the notation to be used, we first consider the standard integral forms of the Navier–Stokes equations of the absolute velocity u_i for an arbitrary moving control volume (e.g., Eqs. 5.13.2 and 5.14.1 of [10])

$$\int [n_i\overbrace{(u_i - w_i)}^{v_i}]d\mathcal{S} = 0, \tag{1}$$

$$\frac{d}{dt}\int u_i d\mathcal{V} = \underbrace{-\int [n_j\overbrace{(u_j - w_j)}^{v_j}u_i]d\mathcal{S} + \frac{1}{\rho}\int(-n_i p + n_j\tau_{ji})d\mathcal{S}}_{\mathrm{RM}_i}. \tag{2}$$

The reference frame for Eqs. (1) and (2) is inertial, and all vectors and tensors are expressed in a Cartesian coordinate system (x_i) in the inertial frame. Cartesian coordinate systems are considered here, because the transformation between the two Cartesian is invariant in space, which is useful in the later derivation. The control volume has a fixed amount of volume $\mathcal{V}$ and face area $\mathcal{S}$. The velocity w_i is the velocity of the control volume with respect to the inertial frame, and v_i is the relative velocity of u_i.

$$v_i = u_i - w_i. \tag{3}$$

[1] via personal conversations with Murali Beddhu.