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FIVER: A finite volume method based on exact two-phase Riemann problems and sparse grids for multi-material flows with large density jumps

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ABSTRACT

A robust finite volume method for the solution of high-speed compressible flows in multi-material domains involving arbitrary equations of state and large density jumps is presented. The global domain of interest can include a moving or deformable subdomain that furthermore may undergo topological changes due to, for example, crack propagation. The key components of the proposed method include: (a) the definition of a discrete surrogate material interface, (b) the computation of a reliable approximation of the fluid state vector on each side of a discrete material interface via the construction and solution of a local, exact, two-phase Riemann problem, (c) the algebraic solution of this auxiliary problem when the equation of state allows it, and (d) the solution of this two-phase Riemann problem using sparse grid tabulations otherwise. The proposed computational method is illustrated with the three-dimensional simulation of the dynamics of an underwater explosion bubble.

1. Introduction

Compressible multi-material flow problems arise in many scientific and engineering applications. These include underwater bubble dynamics associated with explosions and implosions, homogeneous and heterogeneous boiling and bubble collapse phenomena associated with nuclear reactor safety and optimization, and shock wave interactions with material discontinuities such as those characterizing extracorporeal shock wave lithotripsy, to name only a few. The focus of this paper is on a large subset of these problems where the material interface separates regions of pure constituents, and the characteristic size of each region is sufficiently large—for example, much larger than that of bubbles obtained in liquid suspensions—to justify neglecting the effects of surface tension and viscous forces and modeling the flow everywhere by the Euler equations. The focus is also on the class of multi-material flow problems with large density jumps across the material interfaces, say of the order of 1000 or higher. For these problems which arise in many industrial applications including flows of water or molten metals in air, the material interface can be well approximated by a free surface where a fluid on one side of a surface can only apply pressure on the fluid on the other side of that surface. In the high-speed regime and for high

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energy content of the individual constituents, these multi-material flow problems with strong contact discontinuities can also exhibit strong shock and expansion waves.

One approach for solving the class of high-speed compressible inviscid flow problems outlined above is to model them by a multi-material version of the Euler equations and discretize these by a standard conservative shock-capturing scheme. Unfortunately, this approach is known to generate spurious oscillations in the vicinity of the material interface that lead to numerical instabilities (for example, see [1]). Various attempts have been made to address these issues by developing interface capturing methods and specialized discretization schemes near the material interface (for example, see [2–6], and more recently [7]).

Another approach for solving the above class of multi-material flow problems is to explicitly represent the material interface, in view of eliminating the unphysical behavior exhibited by interface capturing methods equipped with Godunov-type [8] discretization schemes. This alternative approach has been developed in three different computational frameworks: Lagrangian, Eulerian, and Arbitrary Lagrangian–Eulerian (ALE). Lagrangian methods move the computational mesh and distort it with the material interface. They convect these interfaces with the local fluid velocity and resolve them sharply by controlling the numerical diffusion around them. Unfortunately, large mesh distorsions induced by large displacements of the material interface can reduce accuracy and numerical stability to the point where a Lagrangian method becomes unpractical. Eulerian methods avoid these issues by adopting a fixed mesh and relying on an auxiliary technique for tracking (or capturing) the free surface. Two well-known examples of such techniques are the volume of fluid (VOF) method [9] equipped with the piecewise-linear interface calculation (PLIC) scheme [10], and the level set method [11] equipped with fast marching methods [12]. ALE methods [13,14]—and their recent extensions known as Reconnection-based ALE methods [15] which allow changes of topology at the rezoning stage—operate directly on the material interface. They advect them by semi-discretizing them on boundary-conforming grids.

Whether the context is set to interface capturing or interface tracking, and in the latter case, whether the Lagrangian, Eulerian, or ALE framework is chosen for performing the computations, it remains to discretize the governing Euler equations at the material interface and solve them. Amid attempts to prevent spurious pressure oscillations and numerical instabilities in multi-material high-speed flow computations, the ghost fluid method (GFM) with the isobaric technique [16] and a related but more economical method known as the ghost fluid method for the poor (GFMP) [17] emerged as simple and elegant alternatives. Both methods were equipped with the level set technique for tracking the material interface. Unfortunately, it was shown that for multi-material flow problems with large density jumps across these interfaces, the GFM and GFMP methods deliver inaccurate results because of spurious oscillations, or simply fail to deliver any result [18,20]. A literature survey suggests that many other solution methods also fail in the presence of large density jumps across the material interface and as a result are limited by some density ratio that is significantly smaller than a realistic value.

To address the issue of strong discontinuities in general, and large density jumps at the material interface in particular, two improved versions of the GFM incorporating approximate and exact two-phase Riemann problems at the material interface, and an improved version of the GFMP method incorporating local, one-dimensional, two-phase Riemann problems at these interfaces were proposed in [18–20], respectively. In particular, the contact preserving method developed in [20] was successfully demonstrated and validated for the three-dimensional simulation of the implosion of an air-filled submerged glass sphere [20]. Unfortunately, all three aforementioned methods appear to be restricted to simple equations of state (EOS) such as those describing perfect and stiffened gases for which fast one-dimensional two-phase Riemann problem solvers can be easily designed. Therefore, the main objective of this paper is to remove this limitation and present a computationally feasible method for the solution of high-speed compressible multi-material inviscid flow problems characterized by large density jumps and arbitrary EOS. To this effect, the remainder of this paper is organized as follows.

In Section 2, the governing equations of the problem of interest are formulated in an Eulerian setting using a general EOS which covers the perfect gas, stiffened gas, and Jones–Wilkins–Lee (JWL) EOS as particular cases, and the chosen finite volume semi-discretization method is specified in general terms. In Section 3, the solution method originally proposed in [20] for the solution of high-speed compressible multi-material flow problems is reviewed and labeled as the finite volume method with exact two-phase Riemann problems as this label describes it best. The advantages of this solution method, such as its suitability for fluid–fluid problems with large density jumps and fluid–structure interaction problems with arbitrarily large deformations and crack propagation, and its shortcomings, such as the effect of a complex EOS on its implementation and CPU performance are also highlighted. To address the latter issue, two different data tabulations pertaining to the solution of an exact two-phase Riemann problem are discussed in Section 4. Then, the concept of efficient interpolation on sparse grids is reviewed in Section 5 to keep this paper as self-contained as possible, and that of data tabulation is tailored in Section 6 to the case of the JWL EOS. Next, the finite volume method with exact two-phase Riemann problems and sparse grids proposed for the solution of multi-material flow problems with large density jumps is evaluated in Section 7 with its application to the three-dimensional simulation of the dynamics of an underwater explosion bubble. Finally, summary and conclusions are given in Section 8.

2. Governing equations

In this work, the Eulerian framework is chosen for formulating all flow problems. This framework is suitable not only for multi-fluid applications, but also for multi-material flow problems involving static or dynamic, rigid or flexible solid