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A large time step 1D upwind explicit scheme (CFL > 1): Application to shallow water equations

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ABSTRACT

It is possible to relax the Courant–Friedrichs–Lewy condition over the time step when using explicit schemes. This method, proposed by Leveque, provides accurate and correct solutions of non-sonic shocks. Rarefactions need some adjustments which are explored in the present work with scalar equation and systems of equations. The non-conservative terms that appear in systems of conservation laws introduce an extra difficulty in practical application. The way to deal with source terms is incorporated into the proposed procedure. The boundary treatment is analysed and a reflection wave technique is considered. In presence of strong discontinuities or important source terms, a strategy is proposed to control the stability of the method allowing the largest time step possible. The performance of the above scheme is evaluated to solve the homogeneous shallow water equations and the shallow water equations with source terms.

1. Introduction

Upwind methods have proved a suitable way to discretize the shallow water equations being able to predict the water profile and discharges in hydraulic modelling [1]. The first order explicit upwind method, in particular, has gained widespread acceptance in this area because of its conceptual simplicity despite the time step size is restricted by stability reasons to fulfil the Courant–Friedrichs–Lewy (CFL) condition.

It is possible to relax the condition over the time step size when using explicit schemes. A generalisation of the first order explicit upwind scheme, modified to allow large time steps, was explored by Leveque [2,3] (large time step, LTS) first in the scalar non-linear case and then adapted to systems of equations. It becomes stable for CFL values larger than one and provides accurate and correct solutions of shocks. Some difficulties can be met when a rarefaction is present in the solution so that adjustments are necessary. Other class of large time step explicit schemes based on TVD properties [4] have been analysed and tested mainly for the scalar equations or systems of equations without source terms. These will not be considered in the present work.

The LTS scheme is increasingly used because it is able to achieve a reduction in the computational time keeping reasonably accurate. Engineering applications related with atmospheric dynamics [5] and Euler equations [6] have been recently published. The shallow water equations, being a hyperbolic system of equations, are also a good candidate for the application of the LTS scheme and an overview of this scheme in the context of the shallow water system was presented in [7]. The source term treatment and the boundary conditions discretization are crucial to allow stability in presence large CFL values in realistic cases.

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The source term discretization has been strongly discussed in the literature. The main focus consisting on maintaining the discrete balance between flux and source terms giving rise to well-balanced schemes [8,9,1] has given way to techniques that prevent instability and ensure conservation by a suitable flux difference redistribution [10] avoiding the necessity of reducing the time step below the CFL condition. The idea of using a stationary jump discontinuity representing the source term in the Riemann solution [11] and the corresponding augmented approximate Riemann solvers for the shallow water equations [12] can be incorporated to the LTS scheme. Moreover, in several situations, the presence of large source terms playing a leading role over the convective terms can lead to wrong solutions using the LTS because the wave celerity is not well estimated due to the reduced number of time steps done. A way of overcoming this situation is also proposed providing the Rankine–Hugoniot conditions derived from the Riemann problem analysis.

The boundary conditions dicretization is another issue of importance in a numerical model. In the context of the shallow water equations, open boundaries and closed boundaries can appear and must be analysed. From the structure of the LTS scheme, information is transmitted not only to the immediate neighbouring cells but also to a number of other cells growing as the CFL value increases. Therefore, some information can cross the boundaries and a careful consideration is required in order to reproduce all kind of scenarios such as subcritical, supercritical and closed boundaries. A first approximation of the boundary treatment was also proposed in [7], where an accumulation technique was suggested in the case of closed boundaries. Another possibility called reflection technique is considered here.

This method is proposed to be a general tool for solving the 1D shallow water equations for open channel and river flow problems. Several problems such as wet/dry fronts, sonic points, changes in the flow regime or large discontinuities are already solved for the conventional upwind explicit scheme hence a kind of CFL limiter can be proposed in order to reduce the initial CFL number or directly recover the original scheme with CFL = 1 when these situations are present.

The outline is as follows: the discretization is described first, for 1D scalar equations with and without source terms. In the non-linear case,the treatment of the rarefaction waves is explored. Then, the scheme is extended to systems of equations, in particular to solve the shallow water equations where bed slope and friction source terms are incorporated into the proposed procedure. The way of dealing with the boundaries is analysed in the cases of systems and two possibilities are proposed: an accumulation technique and a reflection technique. They are tested in a dam break problem with solid wall conditions in the inlet and outlet boundaries. Moreover, the use of a parameter that limits the CFL number in the presence of big discontinuities or large source terms is proposed. Finally, the scheme is evaluated and tested trough several problems with analytical solutions where the bed slope and the friction terms plays a leading role.

2. Scalar equations

2.1. Linear scalar equation

Consider the linear scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0 \tag{1}$$

where u is the conserved variable and $f(u)$ is a linear function, $f(u) = \lambda u,\ \lambda = constant$.

The numerical resolution of (1) by means of the first order upwind finite volume method starts by integrating (1) in a volume Ω.

$$\frac{\partial}{\partial t}\int_\Omega u d\Omega + \int_\Omega \frac{\partial}{\partial x} f(u) d\Omega = 0 \tag{2}$$

where $d\Omega$ denotes the volume boundary.

In the case of a uniform discrete mesh $\Omega = \Delta x$. A cell-centred upwind finite volume method is based on a piecewise constant approximation of the function. Therefore, u and f are uniform per cell and the first integral of (2) can be approximated at cell Ω_i by:

$$\frac{\partial}{\partial t}\int_{\Omega_i} u d\Omega = \frac{u_i^{n+1} - u_i^n}{\Delta t}\Delta x \tag{3}$$

After application of the Gauss theorem to the second integral in (2):

$$\int_\Omega \frac{\partial}{\partial x} f(u) d\Omega = f^*_{i+1/2} - f^*_{i-1/2} \tag{4}$$

where the numerical flux $f^*_{i+1/2}$ can be determined using an approximate solver. The numerical scheme can be formulated in a general way as:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left(f^*_{i+1/2} - f^*_{i-1/2}\right) \tag{5}$$