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An implicit immersed boundary method for three-dimensional fluid–membrane interactions

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ABSTRACT

We present an implicit immersed boundary method for the incompressible Navier–Stokes equations capable of handling three-dimensional membrane–fluid flow interactions. The goal of our approach is to greatly improve the time step by using the Jacobian-free Newton–Krylov method (JFNK) to advance the location of the elastic membrane implicitly. The most attractive feature of this Jacobian-free approach is Newton-like nonlinear convergence without the cost of forming and storing the true Jacobian. The Generalized Minimal Residual method (GMRES), which is a widely used Krylov-subspace iterative method, is used to update the search direction required for each Newton iteration. Each GMRES iteration only requires the action of the Jacobian in the form of matrix–vector products and therefore avoids the need of forming and storing the Jacobian matrix explicitly. Once the location of the boundary is obtained, the elastic forces acting at the discrete nodes of the membrane are computed using a finite element model. We then use the immersed boundary method to calculate the hydrodynamic effects and fluid–structure interaction effects such as membrane deformation. The present scheme has been validated by several examples including an oscillatory membrane initially placed in a still fluid, capsule membranes in shear flows and large deformation of red blood cells subjected to stretching force.

1. Introduction

This paper considers an implicit immersed boundary method for simulating viscous incompressible flows with immersed elastic membranes. The immersed boundary (IB) method was originally developed by Peskin [36] to study the fluid dynamics of blood flow in the human heart. Peskin's immersed boundary method has proven to be a very useful method for modeling fluid–structure interaction involving large geometry variations. The original method has been developed further and applied to many biological problems including platelet aggregation [14,15,52], the deformation of red blood cells in a shear flow [11], the swimming of bacterial organisms and others [10,13]. More details on the immersed boundary method can be found in [37] and the references therein.

Typically, in the framework of the IB method, the elastic boundary is treated as a collection of elastic fibers. Alternatively, the elastic boundary is also modeled as an elastic membrane [11] or a thin shell [17]. These models are used to compute the

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forces acting at the discrete nodes representing the immersed boundary. For most biological tissues, their membranes are stiff and therefore a small perturbation of the boundary can lead to large elastic forces. This causes a severe restriction in time step required to maintain stability of the immersed boundary method. Much effort has been made to analyze the stiffness of the IB method and remove this restriction [18,22,33,47,48]. Several semi-implicit and implicit methods have been developed to alleviate this problem [12,21,31,35,51]. Comparisons of the explicit method and the implicit methods in the context of moving immersed boundaries have been presented in [35,51]. In the context of immersed interface methods (IIM) a quasi-Newton method has been proposed [27–29] to improve time stability. However, for very stiff problems, small time steps are still required. In order to alleviate this problem, an unconditionally stable discretization of the immersed boundary equations has been proposed [34]. In this scheme, an approximate Newton solver is employed to advance the location of the boundary. The Newton method, however, requires a Jacobian matrix which is extremely expensive to compute explicitly for every time step. To avoid forming the Jacobian matrix explicitly, an iterative matrix-free method has been introduced in the context of the immersed continuum method [53]. Another strategy for solving the implicit IB equations involves deriving Schur complement equations by eliminating one or more of the unknown variables [33,35]. Several methods have been employed to solve the Schur complement equations such as fixed point methods, projection methods and Krylov-subspace methods.

Recently, an efficient semi-implicit IB method with arclength–tangent angle formulation has been proposed for two-dimensional Navier–Stokes equations [21]. In this formulation, an unconditionally stable semi-implicit discretization is derived and the small scale decomposition technique [20] is applied to the discretization. This semi-implicit scheme has much better stability property than the explicit scheme and therefore offers a substantial computational cost saving. We note that the stability of this scheme is weaker than the unconditionally stable scheme proposed in [34] because this scheme treats only the leading order term implicitly [21].

In the present paper, an implicit immersed boundary method is presented with vastly improved time step by using the Jacobian-free Newton–Krylov method (JFNK) [24] to advance the location of the elastic membrane implicitly. This matrix-free approach has many advantages. The most attractive is Newton-like nonlinear convergence without the cost of forming and storing the true Jacobian. In this Jacobian-free method, the search direction required for each Newton iteration is updated using the Generalized Minimal Residual method (GMRES) [44], which is a widely used Krylov-subspace iterative method. Each GMRES iteration only requires the action of the Jacobian in the form of matrix–vector products and therefore avoids the need of forming and storing the Jacobian matrix explicitly. The JFNK method has become established in computational fluid dynamics (CFD) to deal with the nonlinear convection term [3,6,16,25]. In this paper, we employ the JFNK method to advance the membrane location implicitly while still approximating the convection term explicitly. Once the location of the boundary is obtained, the elastic forces acting at the discrete nodes of the membrane are computed. In the present work, an elastic membrane model with bending stiffness proposed in [38,42] is employed. In our numerical studies, the immersed boundary method provides the means of calculating the hydrodynamics and fluid–structure interaction effects such as membrane deformation, and the finite element method with membrane model is used to calculate the elastic forces on the membrane.

The remainder of this paper is organized as follow. In Section 2, we describe the governing equations, the immersed boundary algorithm, the discrete model of capsule membranes and the method for advancing the membrane evolution in time. We then present a detailed implementation of the present method in Section 3. In Section 4, some numerical examples are presented and finally, some conclusions are given in Section 5.

2. Numerical methods

2.1. Governing equations

In a three-dimensional bounded domain Ω that contains an enclosed membrane $\Gamma(t)$, we consider the incompressible Navier–Stokes equations formulated in primitive variables, written as

$$\rho(\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}) = -\nabla p + \mu\Delta\boldsymbol{u} + \boldsymbol{F}, \tag{1}$$

$$\nabla\cdot\boldsymbol{u} = 0 \tag{2}$$

with boundary conditions

$$\boldsymbol{u}|_{\partial\Omega} = \boldsymbol{u}_b, \tag{3}$$

where $\boldsymbol{u}$ is the fluid velocity, p is the pressure, ρ and μ are constant density and viscosity of the fluid, respectively. The effect of the membrane $\Gamma(t)$ immersed in the fluid results in a singular force $\boldsymbol{F}$ which has the form

$$\boldsymbol{F}(\boldsymbol{x},t) = \int_{\Gamma(t)} \boldsymbol{f}(q,r,s,t)\delta(\boldsymbol{x}-\boldsymbol{X}(q,r,s,t))\,\mathrm{d}q\,\mathrm{d}r\mathrm{d}s, \tag{4}$$

where (q,r,s) are curvilinear coordinates attached to the membrane at a material point, $\boldsymbol{X}(q,r,s,t)$ is the position at time t in Cartesian coordinates of the material point whose label is (q,r,s), $\boldsymbol{x} = (x,y,z)$ is spatial position, and $\boldsymbol{f}(q,r,s,t)$ is the force